AN INTERESTING REFERENCE AXIS TRANSFORMATION IN THE COMPUTERIZED ECHOGRAPHY

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Abstract: *Echographic examination, also called ultrasonography, is a painless and non-invasive method of diagnosis, which doesn't cause any kind of lesion in the human body, and which has as a main benefit the possibility of being repeated as frequently as necessary. By reflecting the ultrasounds, it can offer an image, three-dimensional included, of the organs which do not contain air and which allow for such a reflection. The practical issues raised by the development of some computerized echography programs led to an interesting analytic geometry problem – a reference axis transformation problem, whose solution is presented in this article.*

Key words: *Echographic, Euler's angle, computerized echography.*

1. INTRODUCTION

Unlike radiography and computer tomography, which use X-rays and, therefore, have restrictions, precautions, contraindications, and sometimes even lead to negative phenomena, the rays used in echography are, from a physical point of view, ultrasound (sounds with such a high frequency that they cannot be heard).

Some devices are portable, i.e. you can perform an echography at a patient's bedside, or even at his/her residence. There are devices equipped with the option of transmitting, receiving and analyzing Doppler signals. These signals can measure the direction and size of the speed in the case of flowing liquids such as the blood, being extremely useful especially in heart echography. Some higher performance devices have the possibility of visualizing structures three-dimensionally.

Echographies have a very reasonable cost if we appreciate their cost based on the ratio between investments and the amount of information supplied.

Specialists believe that the profitability of this diagnosis method has not been exceeded by any other methods. Yet, its limitations consist in the fact that it cannot be applied to the organs which contain air, or which are located behind the bone structures. The echography measures the organs, estimates their shape and architecture, but does not provide information about their physiology.

The method cannot reveal structures smaller than a certain size due to the minimum resolution restriction. Usually, this is valid for structures smaller than 3 mm [1].

2. SYSTEMS OF REFERENCE

Considering a system of reference Oxyz attached to a transmitter S, another system of reference $O_1x_1y_1z_1$ attached to a receiver R_1 and the system $O_2x_2y_2z_2$ attached to a second receiver, denoted by R_2 [1].

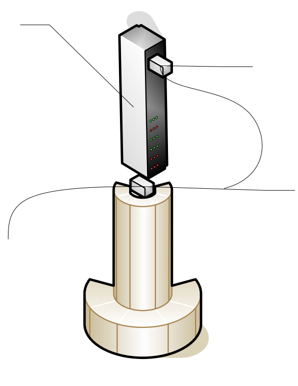

Fig. 1 Fix and mobil sensor

Figure 1 illustrates the positioning of the two sensors, both fixed and mobile, to which the trihedrals of reference denoted by 1, respectively 2 are attached.

The position of the trihedral $O_1x_1y_1z_1$ (or of the trihedral $O_2x_2y_2z_2$) in relation to the system attached to the transmitter is described by way of Euler angles (fig. 2) [1].

Any rotation in space can be described by using three angles. If the rotation is expressed by way of the terms of some rotation matrixes, denoted by B, C and D, then a general rotation A can be expressed:

$$A = BCD \qquad (1)$$

The three angles, which comprise the terms of the rotation matrixes, are Euler angles. There are several conventions for the Euler angles, depending on the rotation axes that are used. Considering the matrix A_{as} [2, 3]:

$$A_{as} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad (2)$$

we will obtain the so-called “x-convention” transformation, illustrated in figure 1, which is also the best known definition [1].

The data recorded with the help of the two sensors are processed according to the scheme in figure 3.

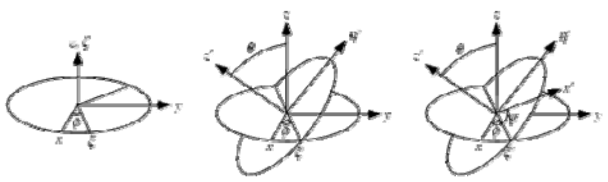

Fig. 2 Euler’s angles

In this convention, the rotation is given by the angles (Φ,θ,ψ), in which the first rotation is with an angle Φ around the axis z, the second one with the angle $\theta \in [0, \pi]$, around the axis x, and the third one with the angle ψ again around the axis z [4].

Thus, in the convention described in figure 1, the rotation matrixes are:

$$D = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

$$C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \qquad (4)$$

$$B = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (5)$$

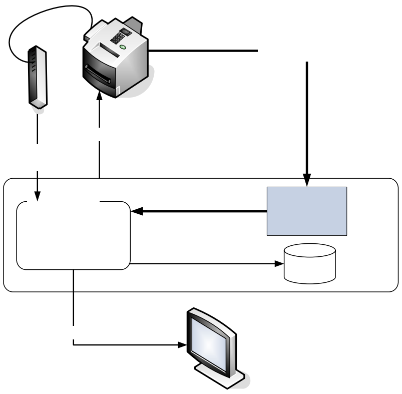

Fig. 3 The data recorded

3. THE TRANSFORMATION OF THE REFERENCE AXIS

As a result of the measurements that are performed, the positions occupied in space by the reference point $O_1x_1y_1z_1$ in relation to the reference point Oxyz, attached to the trasmitter, as well as the positions of the reference point $O_2x_2y_2z_2$ also in relation to the first reference point Oxyz, are known. Therefore, it is required that for each set of six measured values (three distances and three angles), which describes the position of the trihedral $O_1x_1y_1z_1$ in relation to Oxyz and other six values, which describe the position of the trihedral $O_2x_2y_2z_2$ also in relation to the trihedral Oxyz, be calculated a set of six values, which expresses the position of the reference point $O_2x_2y_2z_2$ in relation to $O_1x_1y_1z_1$ [2, 3].

For example, in table 1, a set of discrete values is written, which describes the position of the trihedral 1 (sensor 0) in relation to the trihedral attached to the transmitter, and the position of the trihedral 2 (sensor 1) also in relation to it.

The seventh value is not a coordinate, but it represents a reaction value (the data acquiring time, expressed in miliseconds) [1].

Table 1 A set of discrete values

X Y Z
A (Aziumuth) E (Elevation) R (rotation)
Timestamp (unix format)

Sensor0: 428.541 6.558 34.212 -175.834 0.698 -90.909 1141786547.644
Sensor1: 336.035 -6.251 53.913 3.358 2.058 -1.094 1141786547.644

Sensor0: 342.649 -10.130 -42.332 79.880 -80.584 -173.167 1141786557.450
Sensor1: 323.562 -5.888 35.607 -0.742 4.909 -2.444 1141786557.460

Sensor0: 344.072 -81.344 0.530 95.460 -22.504 -102.207 1141786567.935
Sensor1: 329.310 -4.995 46.462 5.966 3.611 1.011 1141786567.945

Taking into consideration the transformation law, it can be written:

$$X = X_{01} + A_1 X_1 \quad (6)$$

$$X = X_{02} + A_2 X_2 \quad (7)$$

Similarly, between the sensor 1 and the sensor 0:

$$X_2 = X_{12} + A_{21} X_1 \quad (8)$$

If a certain point is selected, for example in the system of reference attached to the transmitter M(1, 1, 1), the coordinates of that point in the system $O_1x_1y_1z_1$ can be calculated from the relation 6.

Then, the coordinates of the same point M in relation to the trihedral $O_2x_2y_2z_2$ can be calculated from the relation 16.

As a result, we can find out the terms of the matrixes X_1 and X_2 from the relation 8, from which the terms of the matrix A_{21} will be calculated, which describe the position searched of the trihedral $O_2x_2y_2z_2$ in relation to the trihedral $O_1x_1y_1z_1$ [1].

For example, if a null translation is considered in the relation (8), it becomes:

$$X_2 = A_{21} X_1 \quad (9)$$

which is also written:

$$\begin{cases} x_1 = a_{11}x_2 + a_{12}y_2 + a_{13}z_2 \\ y_1 = a_{21}x_2 + a_{22}y_2 + a_{23}z_2 \\ z_1 = a_{31}x_2 + a_{32}y_2 + a_{33}z_2 \end{cases} \quad (10)$$

The well-known relations between the coefficients of the transformation matrix (10) are attached to the relations (11):

$$\begin{cases} a_{11}^2 + a_{21}^2 + a_{31}^2 = 1 \\ a_{12}^2 + a_{22}^2 + a_{32}^2 = 1 \\ a_{13}^2 + a_{23}^2 + a_{33}^2 = 1 \\ a_{11}a_{12} + a_{21}a_{22} + a_{31}a_{32} = 0 \\ a_{11}a_{13} + a_{21}a_{23} + a_{31}a_{33} = 0 \\ a_{12}a_{13} + a_{22}a_{23} + a_{32}a_{33} = 0 \end{cases} \quad (11)$$

Which, altogether with the relations (20), form a nonlinear system of 9 equations with 9 variables, the very terms of the matrix A, which express the position of the second sensor in relation to the first sensor, which means resolving the initial transformation problem [1].

$$\begin{cases} x = X\cos Xx + Y\cos Yx + Z\cos Zx \\ y = X\cos Xy + Y\cos Yy + Z\cos Zy \\ z = X\cos Xz + Y\cos Yy + Z\cos Zz \end{cases} \quad (12)$$

If of the systems of reference is oblique - a less common situation encountered in practice, passing from the coordinates of one point, expressed in relation to the oblique system, to the coordinates of the same point, expressed in relation to the rectangular system, can be done by using formulae with the same form (12), except that in this case the cosines satisfy only the first 3 relations from (13):

$$\begin{cases} x = a_{11}X + a_{12}Y + a_{13}Z \\ y = a_{21}X + a_{22}Y + a_{23}Z \\ z = a_{31}X + a_{32}Y + a_{33}Z \end{cases} \quad (13)$$

As an example, table 2 lists a set of 12 discrete positions measured by the first sensor, each one of them being defined by other 6 values, 3 distances (record x, record y, record z), respectively 3 angles, record a, record e, and record r.

In table 3, the 12 positions of the second sensor are listed.

Finally, table 4 contains the calculated discrete values which describe the position of the sensor 2 in relation to the sensor 1.

Table 2 A set of discrete positions measured by the first sensor

Senzor	record.x	record.y	record.z	record.a	record.e	record.r
Sensor0:	426,8390625	-25,003125	44,00661621	-1,840209961	-3,707885742	91,77429199
Sensor0:	426,8390625	-25,11474609	44,06242676	-1,889648438	-3,713378906	91,77429199
Sensor0:	426,8390625	-25,05893555	44,06242676	-1,873168945	-3,71887207	91,78527832
Sensor0:	426,8390625	-25,003125	44,06242676	-1,867675781	-3,707885742	91,78527832
Sensor0:	426,8390625	-25,05893555	44,00661621	-1,900634766	-3,69140625	91,79077148
Sensor0:	426,8390625	-25,003125	44,1182373	-1,889648438	-3,707885742	91,80725098
Sensor0:	426,894873	-25,003125	44,06242676	-1,889648438	-3,696899414	91,80725098
Sensor0:	426,783252	-25,003125	44,1182373	-1,900634766	-3,696899414	91,82922363

Table 3 A set of discrete positions measured by the second sensor

Senzor	record.x	record.y	record.z	record.a	record.e	record.r
Sensor1:	517,8660645	-18,58491211	44,67634277	-178,5443115	0,461425781	87,86315918
Sensor1:	517,8660645	-18,58491211	44,67634277	-178,560791	0,461425781	87,85217285
Sensor1:	517,9776855	-18,58491211	44,67634277	-178,560791	0,461425781	87,85766602
Sensor1:	517,921875	-18,64072266	44,67634277	-178,5772705	0,450439453	87,84667969
Sensor1:	518,0334961	-18,64072266	44,62053223	-178,5772705	0,422973633	87,84667969
Sensor1:	517,9776855	-18,52910156	44,62053223	-178,560791	0,428466797	87,85217285
Sensor1:	518,0334961	-18,52910156	44,62053223	-178,560791	0,428466797	87,85217285
Sensor1:	517,9776855	-18,52910156	44,67634277	-178,560791	0,439453125	87,83569336

Table 4 The calculated discrete values

Senzor	record.x	record.y	record.z	record.a	record.e	record.r
Sensor_ref:	-0,012749795	0,005799272	0,008011126	0,012649853	-0,0026836	-0,012554988
Sensor_ref:	-0,014851343	-0,055624166	0,139581634	0,026599269	0,032299324	-0,020295302
Sensor_ref:	-0,127470752	-0,053912577	0,081201751	0,02312688	0,011303796	-0,009902029
Sensor_ref:	-0,062937961	-0,037232095	-0,000408838	0,02423796	-0,00650701	-0,009927075
Sensor_ref:	-0,178942804	0,004319556	0,053929831	0,038621547	0,020600282	-0,005849072
Sensor_ref:	-0,12715429	-0,118785849	0,078500342	0,050184159	0,03148811	0,010748323
Sensor_ref:	-0,127787757	-0,057460968	0,080696105	0,043214051	0,031466341	0,007748224
Sensor_ref:	-0,182974949	-0,08127524	0,078861642	0,028147299	0,039150264	0,021752368
Sensor_ref:	-0,064709655	-0,062225641	0,043354351	0,021058439	0,031971041	0,038119751
Sensor_ref:	-0,06470101	-0,062242812	0,043345795	0,031838792	0,035439239	0,049792027
Sensor_ref:	-0,012142991	-0,110132283	0,125920248	0,034906297	0,031742167	0,046517023

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