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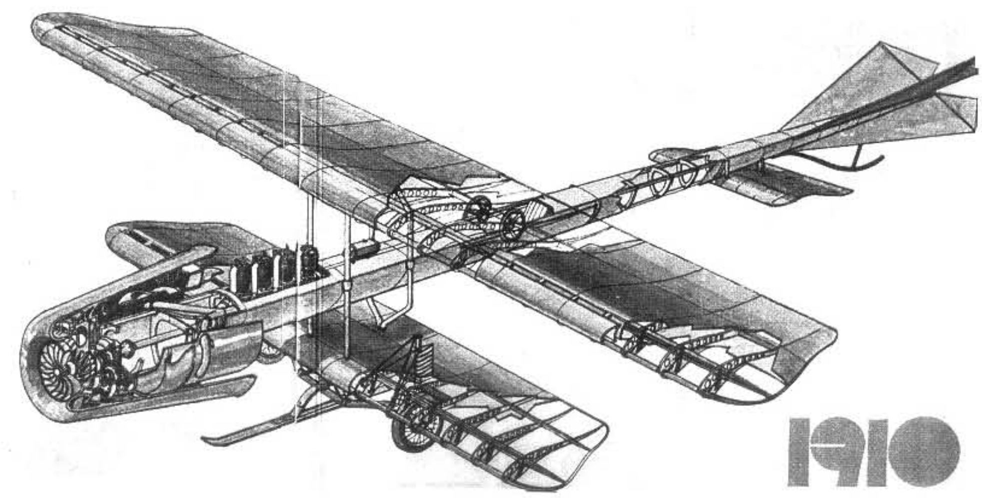

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C O N T E N T S

THE LEADERS' COURAGE TO ACHIEVE PERFORMANCE STARTS FROM FOLLOWING THE DICTATES OF CONSCIENCE

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Abstract: *Conscience represents the feeling of duty, love and compassion that a leader feels towards all that is life, according to the absolute knowledge of good and evil, as a goal at which he/she aims full of hope, according to his/her free will, i.e. the ability to choose good over evil based at one's own will.*

Keywords: *conscience/ leader/ leadership/ authority*

1. GENERAL INFORMATION ABOUT LEADERS AND PERFORMANCE IN MANAGEMENT

While studying performance in a certain field of activity, the psychologist Anders Ericson formulated the empiric rule of the 10,000 hours, which specifies that "You don't get benefits from mechanical repetition, but by adjusting your execution over and over to get closer to your goal." By raising the stake and accepting the possibility of a large number of errors occurring at the beginning, as the horizons extend, almost all of us can achieve the highest level of performance by means of intelligent training, i.e. "deliberate education", where an experienced leader includes his/her most involved peers in well-thought-out education systems for months or even years on end.

The exercise of applying empiricism to management for 10,000 hours certainly cannot create leaders, but only versed Machiavellian people.

Repetitive exercise for hours on end is not enough. The difference is made by the focus on constructive feedback that enables the recognition of errors and the transfer/discovery of the corrective measures. The same as ballet dancers need mirrors when performance is pursued in dancing, performance requires constructive feedback in any other field of activity.

Apart from the persistent training focused on constructive feedback, in order to achieve performance disciplined attention is also required, i.e. the capacity to observe one's own mind in order to determine when it begins to wander away and to bring it back to/fix it on the positive side of the point of interest by relinquishing the egocentric "I" and replacing it with the more friendly "we". Bearing in mind the stake, the effort is worth it, i.e. the neuronal network of the brain that corresponds to the value of the compensatory system, the neuronal network that determines motivation, more precisely the rewards for the efforts made. As long as we maintain a positive attitude, our neuronal systems are rich in hormones (dopamine, endorphins, serotonin) that generate good mood and stimulate perseverance and the achievement of the set and desired goals, thus feeding our will and giving us some sort of pleasure.

Leaders and the 7th sense. Leaders are brave persons. The courage of leaders, which is extremely precious, particularly in dire situations, is based on the spirituality of the evolved human beings who let their conscience be their guide while remaining faithful to their emotional attachments (love/ compassion) to everything that represents is life and being willing to sacrifice their personal needs if they endanger any of their fellow men or life in general.

Why is this human typology so rarely seen in the history of mankind and especially nowadays? Where has this courage of sacrifice for a noble cause such as life in any of its forms gone?

It seems that we are equipped with 7 senses: eyesight, hearing, touch, taste, smell, intuition and, last but not least, conscience. The last two are not at all exercised by the majority of human beings, whereas leaders are capable of achieving their entire sensitive potential by living fully and assuming their mission of human beings.

Currently, moral leadership represents the capacity of certain individuals to approach the problems generated by the lack of resources and insecurity in a pragmatic manner, by renouncing to temptation to turn another group than the one they belong to into a scapegoat.

Unfortunately, history has shown us that there are also anti-leaders, i.e. individuals who have no conscience (4% of the world's population is represented by sociopaths), who have the ability to hypnotise entire groups of people and drive society towards catastrophe. Why are anti-leaders more charismatic? The trance state of the conscience of anti-leaders makes them believe that their peers are objects and that is only due to the fact that they hold the authority to induce fear and helplessness with the aim to annihilate their peers' moral compass. The tendency to reduce people to non-beings is unconscious and it represents the basis for a cruel treatment of innocent people that have been downgraded to the status of "not even people", i.e. objects, at a certain point in history by a part of humanity. The list of these victims is shamefully long and it includes all the possible types of categories: black people, communists, homosexuals, Jews, witches, women, Muslims, Christians, Palestinians, poor people, rich people, Irish, English, Albanese, Croatian, Serbian etc. In dire situations, i.e. during acts of torture, war, genocide or economic crisis, individual conscience enters a state of deep trance and the role of leaders as people who are equipped with the 7th sense, i.e. with conscience, is extremely important, because they have the courage to make the difference between the gradual awakening of the conscience of masses and the endless nightmare of amorality.

What should be done? When is it moral to oppose the authority of anti-leaders? What cannot authority impose on us? They shouldn't ask us to cause suffering to helpless people, who are not dangerous or threatening to anyone. Based on the education received, any individual can build his/her own perception of himself/herself as a being with legitimate authority. It seems that 38% of the world population complies with and answers only to moral authority, while the other 62% of the individuals answer to authority in general, thus losing contact with their own conscience. In other words, the 32% of the population are brave enough to follow their own conscience, will act in a moral way and will oppose immoral authority. They will refuse to provoke suffering to helpless people who represent no danger/threat to the people around them.

The challenges a leader faces stem from the fact that he/she follows his/her own conscience and they include:

- the fact that the leader shall take full responsibility for his/her own actions;

- the fact that the leader does not stand out from the mass of peers, from people who lack conscience (apart from charisma, sociopaths also have the capacity to dissimulate moral behaviour, which is the psychological defense mechanism used by sociopaths, this being their ability to become the opposite of authority);

The premises for recognising the difference between a leader and an anti-leader are the following:

1. There’s no image of evil;
2. The individual should not be assigned the integrity of the role (the qualities of the role);
3. We shouldn’t let the manner in which a person presents himself/herself as benevolent, creative or intuitive distract us from observing their real behaviour; we should only believe what we see, not what that person says about himself/herself;
4. We should recognise when fundamentally positive tools that are used against us for the purpose of holding society together. Among them are empathic feelings, sexual relations, professional/social roles, consideration for the merciful/creative people, the desire to make the world a better place, demagogy;
5. We should recognise the lack of consideration for social contract combined with the use of the contract for one’s own advantage, while acknowledging the negative/shamelessly inadequate behaviour with some moments of begging for pity.

2. THE LEADER’S TRUE SELF

The analysis of the self involves bringing to consciousness the aspects that the individual is yet unaware of and the beliefs that are mostly built on his/her preferences for certain empathic answers from his/her genitors.

The true self involves passing from aspirations to the most vital power of early personal grandeur. This is the ideal that could vary from one individual to another, but that remains, in essence, the same, i.e. the feeling of relaxed, peaceful fulfilment that one feels when the layers of the nuclear self are truly involved in the undiscovered and unbuilt structures that are important for the deep layers of the personality [10].

Early crucial experiences, where a certain balanced pride connects with the enthusiasm related to various forms of greatness of others (money, power, kindness, generosity, altruism, etc.), form cohesive considerations and the conscience of the leader. The dedication to such an individual self enables leaders to manifest the type of courage that determines them to consciously risk their own physical destruction for something more important than the survival of the nuclear self through devotion to the ideal. Leaders have multiple selves, which is not a disease or a split personality [10]. They can take advantage of various cohesive forms of themselves that exist at the surface and in depth and that are independent from one another. Furthermore, in the creative leaders there is a tension between two different types of commitments (for example two types of education that make them separate the two types of self throughout their entire lives, without choosing between the two and even gaining an advantage from this). The more central position shall be held by the type of self that is closest to the centre through its quality of point of convergence of the leader’s most profound ideals [10].

The leader’s healthy self has the insight capacity, i.e. the capacity to courageously self-analyse his/her experiences, consisting of the leader’s grand ideas that become his/her main ambitions in life, as well as the most in-depth type of narcissist identification with some idealised, omnipotent figure that he/she identifies himself/herself with (the divinity), which turns into his/her guiding goals, values and objectives in life.

It also involves the functions fulfilled by means of the specific inborn talents that get (or only operate randomly) to be more intensely mobilised and stimulated by the tensions that occur between ideals, such as grand ambitions, and the idealised goals. Thus, an arch of energy is born during the development, through tensions that occur between one's ideality and talents. Once born, it shall remain present throughout the leader's entire life as a healthy individual. This energetic spectrum or tension arch formed early in life, shall forever shape the unfolding events in the leader's life, thus representing the fundamental expression of the nuclear self as a mixture of:

- the leader's fundamental ambitions;
- the person's fundamental goals;
- the functions of the ego that mediate between the two poles (ambitions and goals).

This mixture represents the mental and physical unity in time and space and it consists of the fact that the individual has a past, a future and some sort of uniformity throughout his/her life. Uniformity is built on the immutable foundation of the ego's most basic ambitions that characterise this energetic spectrum. Herein under are the factors that give strength and weakness to the nuclear self:

- the balanced distribution of the ambitions of self-expression (honesty, decisiveness, realism, fairness) in agreement with the idealist preoccupations (to save/support life, love, happiness) by means of properly functional activities of the self and the individual's talents that help him/her express his/her ambitions harmoniously and as easily/efficiently as possible for idealist purposes.
- The fundamental, reflective objects of the self (people inside or outside the parental environment that have the capacity to empathically accept high values and receptivity by encouraging healthy exhibitionist or idealised attitudes and by investing a little bit in the narcissism related to the ambitions and affirmation of the child who shall later become a leader.

The anti-leader is a person that is defined by fragile self-cohesion, i.e. this cohesion depends on favourable external circumstances and it tends to disintegrate when there are no narcissist resources (support, praise or approval) [10]. More precisely, when the idealised object does not allow the close connection required for the merger or fusion to take place, the grandiose self lacks vigour (the signs of disturbance of the self are vague fears, cold and arrogant grandeur, conventional ways of thinking, neologisms and mannerism, a certain insecurity, depression, lack of vitality, lack of initiative).The narcissist anger, which consists of various acute and chronic reactions, such as more or less violent temper tantrums, bitterness, sarcasm, irascibility, arrogance, criticism, are caused by the anti-leader's inability to affirm himself/herself as he/she is, because of the fear of his/her own vulnerability if he/she were to support him/her claims in a resolute and calm manner. This is related to the sensitivity of an individual who hides his/her need to assert himself/herself, who confuses the past with the present and who transfers his/her childhood frustrations caused by an unloving and indifferent mother [10].

In stressful situations, the precarious self of the narcissist anti-leader shall disintegrate, meaning that it shall manifest in fragmented forms, such as: hypochondriac preoccupations, regressive oscillations between self-esteem, arrogant attitudes, weird behaviour, insufficient coherence, clothing disorder, depression, a feeling of emptiness, lack of purpose, hopelessness, depression. All of these manifestations of a fragmented self are not characterised by culpability, but rather by feelings of emptiness, lack of meaning and the loss of vitality. The self can reintegrate itself only after a proper reinterpretation, i.e. the acknowledgement of loss, regardless of the type: lack of parental empathy/attention early in life, lack of support caused by defective empathy, that is sometimes very strong and other times very weak, an imbalanced empathy that causes hypersensitivity.

3. CONCLUSION: THE ACTION PRECEEDED BY THE METHOD OF CONCENTRATION ON ONE'S OWN CONCIENCE

Regardless of whether we try to reach performance in sports, music or management, the essential elements of the training process remain the same:

- positive attitude
- inspired strategy
- maximum of attention.

Having a conscience involves the effort made by the evolved individual to raise at the level of his/her human side, which entails focussing on the following three directions:

1. Internal attention (intuition)
2. Attention to others (empathy)
3. External attention (general view)

Korea and other Asian states consider that internet addiction, together with gambling, social media and virtual reality represent a national health crisis that leads to the isolation of the individuals due to the loss of attention to others, as well as the loss of internal and external attention (Daniel Goleman, *Focus-Motivația asupra performanței [Focus – Motivation over performance]*, Curtea Veche Publishing House, 2014, page 18).

Around 8% of the American population aged 8 to 18 that gamble, meet the diagnosis criteria for addiction, which are very similar to the ones used in identifying alcohol and drug addicts. The lack of attention in adults is visible in their incapacity to read more than two pages at a time, in the fact that advertisements must be reduced to one minute and a half only to capture the attention of the targeted public and in the indivduals' incapacity to concentrate on the dialogues with other people. "Partial constant attention" is a mental obscurity induced by the overloading with information from other people, plus what we do on our own computers and whatever else we do simultaneously (talking on the phone, reading text messages).

The individual's power of concentration is in a constant battle with all the other inner and outer distractions.

William James, one of the founders of modern psychology, defined attention as "is the taking possession by the mind, in clear and vivid form, of one out of what may seem several simultaneously possible objects or trains of thought." There are two types of factors that distract attention:

- Sensorial;
- Emotional.

When leaders, as healthy individuals, concentrate on something, they ignore the sensorial factors that distract attention (breathing, different smells, sounds, shapes, colours, tastes).

The people who manage to concentrate successfully are relatively immune to emotional distraction (emotional factors), thus remaining imperturbable in crisis situations and being able to move on and achieve balance, despite the emotional waves that they are confronted with in life.

When pushed to the extreme, leaving one's mind captive in the same repetitive circles of chronic anxiety is the same as feeling sorry for oneself and giving in to depression, hopelessness and worthlessness, or panicking and acting in the middle of a state of anxiety, or having recurrent superstitious thoughts and reactions, thus becoming the victim of some obsessive-compulsive disorder.

One's state of wellbeing depends on the power to disengage our attention from one thing and to direct it towards another. The more selectively attentive we are, the more intensely we continue to be absorbed by what we have chosen to do.

Concentration makes people abandon themselves and forget about turmoil of their daily lives. Focused people can be easily recognised by their capacity to sink deeply into a conversation, despite the noises that surround them, looking straight into the eyes of the interlocutor and being absorbed by the intensity of their own words.

During intense concentration, the circuits in the prefrontal cortex synchronise with the object of the awareness radar and this is called "a state of fixation". The higher the level of concentration, the deeper the state of neuronal fixation.

At the opposite pole we have people who cannot concentrate and who are in a constant state of restlessness, looking around in search of something that could arise their interest, thus having dispersed attention. These people have an attention deficit, i.e. a reduced level of synchronisation (chaotic and disorganised thoughts).

During the learning process, people focus their attention on what they are studying. Their brain maps the information that it already knows, thus generating new neuronal connexions. The synchronisation between an idea and a sound is called fixation. The essence of learning is represented by the fact that the mind builds a mental pattern that helps us understand what we are reading and connects this understanding with the universe of patterns that we already possess in relation to that particular topic. Deep thought requires us to maintain a focused unity. The danger of learning being obstructed by distracting elements related to multimedia (simply called the INTERNET) is imminent. In the mid-20th century, philosopher Martin Heidegger stated that "the looming tide of the technological revolution" might so "captivate, bewitch, dazzle and beguile mankind that calculative thinking becomes the only way of thinking", to the detriment of "reflective, contemplative thinking", which is the essence of humanity, the foundation of reflection and the capacity to maintain focus on a continuous narrative thread. A profound mind is a focused mind. Leaders are people with a high capacity of concentration, which is fundamental to their motivation and the reason why they love what they do and why they feel so good, pleasure being the personal attribute for "the state of fluidity". 80% of people are either stressed or bored, while 20% of people lose their concentration at least once a day. Leaders have the ability to concentrate more than the majority of the population due to their authenticity (their capacity to combine what they do with what they love to do), as well as the fact that they build their self-confidence and self-esteem on fulfilling challenging tasks that require them to nake the best of their abilities and to carry out activities that they are passionate about and that generate strong satisfaction.

Nowadays, more and more individuals are stuck in a state of "burning", in which the constant stress overloads their nervous system with waves of cortisol and adrenaline or with lack of engagement and indifference that reach higher and higher levels and loss of purpose.

The concentration exercise involves ignoring everything else apart from the object of our attention, thus creating a constant tension in a large dichotomic neuronal mechanism, where the upper part of the brain is in a constant clinch with the region located at the base of the brain. Leaders are able to record a large number of details that pass through their minds like lightning owing to their open concentration, thus embracing more feelings, sensations and thoughts, without any emotional reactivity, without fixating on annoying things, without focusing on upsetting details, without feeling anger or any other negative emotions. The ability to maintain the attention wide open by means of panoramic awareness enables leaders to maintain self-control, without being led into a trap that would get their attention from the bottom up and without any positive or negative reactions.

3.1 Bottom-up awareness. Most people practice bottom-up awareness almost all the time, which turns them into innocent victims who find themselves at the mercy of the forces that trigger the action of the subconscious [9]. The cerebellum is the area of the brain that constitutes the neuronal equipment that practices bottom-up awareness. Bottom-up thinking is faster (it operates within milliseconds), automatic (involuntary), intuitive (it operated on networks of association) and impulsive, because it is led by emotions. Moreover, it represents a routine perfomer and an action guide for the individual who performs the actions, as well as a manager for the individual's world-related mental patterns.

3.2 Top-down awareness. Open consciousness is represented by top-down awareness. It is slower, voluntary and spontaneous and it involves a considerable effort on the part of the individual because it is the centre of self-control, which can sometimes put a hold on certain automatic routines, silencing emotionally-drawn impulses, being able to learn new patterns, to make new plans and assuming part of the responsibility for the spontaneous repertoire of the leader.

Leaders practice top-down awareness to a certain extent, as opposed to the majority of people, who practice bottom-up thinking. Keeping one's mind focused on neutral targets (reflective practices based on breathing, touch, taste, rhythmic movement or simply on uttering a request) leads to the completely positive absorption and cease of rumination (disorderly thinking).

The neocortex is the part of the brain that monitors the subcortical equipment and imposes its aims, by practising top-down thinking. The types of attention required for the self-consciousness of leaders are:

-selective attention (focusing on the target and ignoring everything else; it is applied in meditation techniques for purposes of energetic recharge);

-open attention (recording complex information from the environment or from within, thus enabling intuition by choosing subtle signals from the ones received during the wandering of the mind)

Restoring (selective/open) attention is needed to maintain the efficiency of the individual's concentration. This is achieved by suspending thinking during pleasant, recreational activities (strolls, sports, dance, music auditions, reading, drawing, praying etc.).

An optimal executive function is based on the two types of attention, i.e. selective and open, which enable us to assimilate complex information from the environment and from within ourselves, by choosing subtle signals, called intuitions, which we would otherwise ignore.

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LEADERSHIP COMPETENCIES REQUIRED BY THE LEADER IN THE FUTURE OF THE ROMANIAN AIR FORCE

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Abstract: *The duty of each member of the air force is to do everything possible to improve and at the same time develop professionally so that, throughout the entire military career, he or she can evolve and occupy, successively, the full range of positions and specific functions, from the lowest, specific to the graduates of educational institutions to the highest level, and it is the duty of officers to plan this, being the heads of the service branch.*

Keywords: *leadership, experience, education, leadership competencies, leader professional development.*

1. INTRODUCTION

As a first challenge encountered in the process of evaluating the performances obtained by the leaders of any organization, in order to establish the ways to be followed for their further development, is to determine, precisely, the roles, functions or activities performed by them to results in the success of the organization. The next challenge, in order to achieve this endeavor, is to design and build the program, to select the tools and to provide the necessary environment to support the leaders in fulfilling these functions.

The development of leaders, the increase in their performances is all the more important in the current context of volatility, ambiguity and uncertainty, created largely by the changes made both at the global level and within the military organization.

The discussions in this regard should be channeled to the practical side of this activity, and one of the questions we must ask ourselves is "*How can we develop leaders to ensure effective leadership?*".

2. IN SEARCH FOR A NEW SET OF LEADERSHIP COMPETENCIES

From the starting point, represented by the access to the military organization, with the graduation of the Air Force Academy "Henri Coandă" and obtaining the rank of second lieutenant, the development of the acquired competencies, in conjunction with the improvement of the necessary knowledge, is the obligatory way to reach the level of competence imposed by the top level of the military organization. But knowing the road is not enough. The way in which these objectives can be achieved must be considered and several aspects must be considered for this purpose.

One of these has to do with the desire for self-improvement of leaders, with the need to satisfy this, so-called, *thirst for knowledge*, achieved either through formal means or through informal or non-formal means.

If, from an informal point of view, the leaders will have to make use of everything they have learned and sedimented until then, from a formal point of view, the military organization has the obligation to facilitate the creation and the proper functioning of an institutional learning framework, in which officers, from the lowest to the highest rank, can be refined, in order to acquire the competencies necessary to successfully fulfill the roles assigned by the job description.

Knowing the starting point, represented by the accession in the military career, graduating the Air Force Academy courses and obtaining the rank of second lieutenant, but also the finality, occupying suitable functions for the positions at the top levels of the air forces, can help us more easily in the process, pencils of the ways to be followed, for a harmonious professional development, and in accordance with the occupational requirements foreseen to be satisfied.

Having outlined such a road, with clearly established stopping points, it is up to the educational institutions to determine what needs to happen within these *stops*, which are the contents that need to be transmitted but especially what are the methods to be used to maximize the effects imposed through graduate models.

Starting the process of drawing up the model of the graduate, by the categories of forces, has as triggering agent the change of some of the requirements of the beneficiaries, regarding certain behaviors that wish to be observed in their activity.

This is usually done in the annual meetings organized by the military educational institutions, attended by representatives of the military educational institutions, the category of forces and the main beneficiaries, the operational units, working groups whose main purpose is harmonization of the study programs, by agreeing on the contents taught and, if necessary, revising the graduate model.

As a rule, the proposals on the line of the graduate model are submitted by the representatives of the territory of the beneficiary units, because they are the ones who *feel the best pulse*, the need to train the subordinates.

Somewhat in contrast to the ones presented above, as representatives of the military educational institution, we wish to meet the needs of the operational institutions of the air forces and to propose, in this regard, some amendments, regarding the leadership competence described in the model of the graduate. These new amendments are closely linked to the identification of those competencies needed for the leader of the future air forces.

Following a previous study, entitled "*Characteristics that the military leader should possess in the light of the challenges of the future of the Romanian Air Force*" [1], the portrait of the leader of the future of the air forces was drawn. The image obtained was the result of the analysis and summation of the characteristics considered to be suitable by the young officers, fresh graduates of the Air Force Academy "Henry Coandă", and therefore without experience of leading the military structures [2].

The experience gained in the military career, from the point of view of the management, in conjunction with the education, which is achieved during it, are two of the pillars of successful leadership [3]. One way to benefit from them is to consult those who have acquired such qualities, and from the point of view of identifying leadership competencies, the commanders of the units and the large units within the service branch are most able to appreciate our proposals.

Thus, we thought it would be interesting to see if it is possible to identify, at the level of the air forces, a vision regarding what the leadership competencies needed for the leader in the future of this category of forces mean. The common vision, an essential element of leadership, must be shared, on the one hand, by the commanders of the military structures in the Romanian Air Force organization, as direct beneficiaries, and on the other, by the leadership of this service branch, as a deciding factor on the graduate model.

Most leadership application frameworks provide their own definitions of adopted leadership competencies and their components for a more in-depth understanding of these concepts. The simple definition of a competence, however, like the variety of definitions given to leadership, does nothing but present the same thing but with a new set of words. What is really important is how that competence manifests itself within the military organization. The more concrete and concise the description of the actions and behaviors associated with that competence, the more likely it is that this competence will be accepted, understood and, above all, demonstrated.

For the Romanian Air Force, a model of leadership development should be built around the one applied by the United States Air Forces, given the uniqueness conferred to this service branch by the mission entrusted [4]. Providing a roadmap, an orientation map of the leaders of the Romanian Air Force is a key element in achieving success in leadership within this military organization.

For this purpose, as specific objectives in the research project initiated, we set out, as desired, to identify the leadership competencies considered appropriate to be adopted according to the model to be followed by the leaders from the upper level of the Romanian Air Force, to identify the competencies of leadership considered inappropriate for including in the model, completing or revising the examples of behavior used to pencil the model, with the suggestions and recommendations made by the study participants and, as a finality, formulating a set of leadership competencies that could be used as a standard for graduate model development.

In order to validate the competencies, which we proposed to be adopted in the future, we considered that the most appropriate way is to intervene directly with the beneficiary, by consulting the decision elements in the operational units of the air forces.

The choice of subjects, for participating in this consultation, was given not only by the position and nature of the position held, but also by the specialized expertise held in the field of air forces. In addition to these considerations, perhaps the most important quality and the one that brought weight to the tilt of the balance for the selection of the interlocutors, was that of the vast experience in leadership, accumulated throughout the career within the military organization. The term of *extensive leadership experience* we consider that it can define the seniority in the military career of at least 20 years.

Thus, a total number of 75 leadership positions were targeted for interviewing, from group level to service branch ranks [5]. The only exception to this rule was made for the Air Force Training Center for Aviation, the Air Force Training Center for Surface- Based Air Defence, and the Air Force Training Center for Air Surveillance and Electronic Warfare, in terms of their status as institutions for military education, directly involved in the professional development process. As a percentage, the situation is highlighted in FIG. 1.

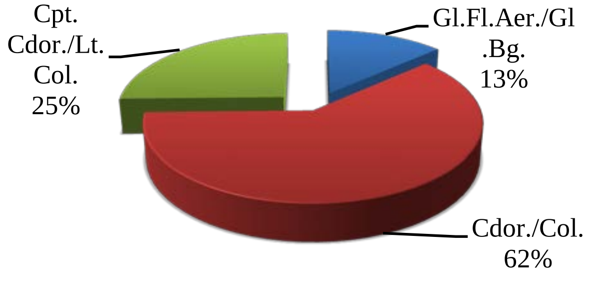

FIG. 1 The degree of the functions of the survey

In the case of the initiated study, the hypothesis of departure was a simple one, and common at the same time, of a confirmatory type.

This was the initial point of the findings of previous research, regarding the development of leadership competencies at the level of the armed forces of the United States of America and the possibility of adopting a similar one, for use within the Romanian Air Force.

There can be two approaches in deducing working hypotheses: deduction from theory and direct experience. As the theoretical basis regarding the competencies needed for air force leaders is reduced, we have resorted to personal experiences in elaborating the following working hypotheses:

1. The degree of consensus achieved by each competence, separately, in order to be adopted and used within the standard, will have to be 100%;
2. The level of acceptance of the proposed competencies is a relevant one and will highlight the role of experience and education in recognizing the theoretical aspects regarding the development of leadership competencies and the need to apply them within the development of the leaders of the Romanian Air Force.

The investigative method used to carry out this study was that of the questionnaire. In the realized approach, a questionnaire was prepared, built on three levels, corresponding to the three levels of competences, with a total of 25 items.

The initiation of the procedure involved the addressing of a letter of intent that would clarify, quickly and succinctly, the necessity and usefulness of this intervention. The purpose of this letter was twofold: to attract attention but also to motivate them to get involved in this project.

The participants were asked to tick with "X" in the box corresponding to the answer of YES or NO for each of the 25 sub-competencies. Marking a YES or NO answer should highlight whether or not the respondents agree that that competence is deemed necessary for the leader at the top level of the air forces.

At the time of applying the questionnaire, its content was, from our point of view, an easy one to follow, precisely by the simplicity of the preparation and especially by the examples of behaviors, which come as a complement, from the need for a better understanding of the competencies sustained.

The final part of this questionnaire was dedicated to the free expression of the interlocutors, by the invitation to complete or to bring to the attention new aspects, which may not be touched by our formulations.

The questionnaire was delivered in two ways: directly, by applying it in print format, or indirectly, through the military information network INTRAMAN, in electronic format. The subjects of the questionnaire were interviewed, for consultation, between 01.03.2019 and 01.05.2019.

The data were collected by simply counting the answers with YES or NOT accounted for in the questionnaire, and were interpreted statistically, while the degree of consensus or number of repetitions should be followed in case of identifying the same answer to several respondents.

From the total number of 75 respondents concerned, we were able to collect answers from a number of 54. This was due, on the one hand, to the lack of the personnel who occupy the respective functions (15 cases), for different reasons (missions, holidays , secondments to courses etc.), or, on the other hand, due to the impossibility of making contact with the respective respondent (4 cases). There were also situations in which the respondents did not accept the participation, motivating the lack of time required to study the material (2 cases). Comments were recorded in 20% of cases, as suggested by point no. IV from the questionnaire.

In the analysis of the collected data, the aim was to identify those elements that would describe the degree of confirmation / rejection of the working hypotheses.

The choice by the respondents, within an item, of the answer NO, was one of free choice and was not subject to a subsequent request for explanatory explanations. It is worth mentioning that during the survey we did not provide detailed information regarding the previous study, which led to the formulation of these proposals. Table 1 shows the situation of the answers collected from the respondents.

Table 1 Level of acceptance of competencies

COMPETENCY		YES	NO
1	Ethical Leadership	54	0
2	Followership	53	1
3	Warrior Ethos	53	1
4	Develops Self	54	0
5	Speaking and Writing	53	1
6	Active Listening	54	0
7	Develops and Inspires Others	54	0
8	Takes Care of People	54	0
9	Diversity	54	0
10	Builds Teams and Coalitions	53	1
11	Negotiating	49	5
12	Operational and Strategic Art	54	0
13	Leverage Technology	53	1
14	Unit, Air Force, Joint, and Coalition Capabilities	51	3
15	Non-adversarial Crisis Response	47	7
16	Enterprise Structure and Relationships	54	0
17	Government Organization and Processes	50	4
18	Global, Regional, and Cultural Awareness	54	0
19	Strategic Communication	43	11
20	Resource Stewardship	54	0
21	Change Management	53	1
22	Continuous Improvement	54	0
23	Vision	53	1
24	Decision-making	53	1
25	Adaptability	53	1

The degree of consensus achieved by each competence, separately, in order to be adopted and used within the standard, must be 100%. In our case it can be observed that a number of 11 competencies have reached the maximum acceptance, and therefore, according to the first working hypothesis, they have qualified for use in the development model. It can also be observed that there were a number of 9 competences that obtained a single negation, a competence with 3, one with 4, one with 5 and one with 11.

If in the case of those who have obtained a single negation we can consider them as isolated cases, individual opinions, which have not been validated by several choices, in the case represented by *Strategic Communication*, the level of rejection (11) is high, in the *Negotiate* (5), *Governmental Organizations* and *Political Processes* (4) and *Unit-level Capabilities, Air Force, Joint Forces and Coalition* (3) is a larger one, but which confirms the existence of a validation of several respondents, aspect of not accepted for choice within the model.

In this context, we considered that the choice of those competencies that registered a single negation, for use within the model, fall within the limitations imposed by the first working hypothesis, and we considered it as an acceptable margin of error.

Regarding the achievement of the degree of consensus, out of a total of 54 respondents, for 41 of them we registered its maximum value, and for the rest of 13 we recorded degrees, as values, in the margin of 72-92%. The consensus obtained is, in our opinion, a good thing in terms of the coherent line between the management factors of the operational units within this service branch and denotes a unity regarding the vision in what is the continuous development of leaders.

Experienced and educated leaders recognize what competencies need to be developed for the future. The collected answers confirm their knowledge of the theoretical aspects regarding the development of leadership competencies and their application in other military organizations and in this sense we could conclude that the second working hypothesis was also confirmed.

The analysis of the recorded comments reinforced our initial conviction regarding the theoretical knowledge of the *leadership phenomenon* by the commanders, and increase the confidence that these, in addition to the practical support of the subordinates development, can be very important factors in the educational process of improving the competencies within the leadership career courses in the Air Force Training School.

In this regard, we consider that the model that can be used as a standard for designing the professional development of air force leaders can be shaped around the 20 leadership competencies that have passed the test of opprobrium and can be presented in the form of three major categories:

1. Personal competency: Embodies Airman Culture (Ethical Leadership, Followership, Warrior Ethos, Develops Self), Communicating (Speaking and Writing, Active Listening);
2. Team competency: Leading People (Develops and Inspires Others, Takes Care of People, Diversity); Fostering Collaborative Relationships (Builds Teams and Coalitions);
3. Organizaţional competency: Employing Military Capabilities (Operational and Strategic Art, Leverage Technology), Enterprise Perspective (Enterprise Structure and Relationships, Global, Regional, and Cultural Awareness), Managing Organizations and Resources (Resource Stewardship, Change Management, Continuous Improvement), Strategic Thinking (Vision, Decision-making, Adaptability).

3. CONCLUSIONS

The most important considerations in developing and establishing leadership competencies should be addressed to how they will be used to influence the processes of leadership evaluation, selection, development and performance management. Even the best leadership application framework has no value unless it is used productively by the organization.

In this way, we consider that the framework model of leadership competencies that resulted from the application of the questionnaire is one that could be used as a standard for designing the professional development of air force leaders. Having thus formulated the development framework, in the future, a program for improving the leadership competencies for officers, starting from the bottom, the rank of first leutenant, and ending with the general, can be outlined.

The starting point of this route must be located when entering the military career, upon admission to the "Henry Coandă" Air Force Academy in Brasov, and will have to include indications regarding both the personal development and the monitoring methods of the progress.

Explaining the competencies in the development model will help leaders determine how they should adjust their behavior. At the same time, explaining the competencies will also help other members of the organization, those with responsibility for developing leaders, to understand, develop or strengthen these behaviors.

A possible image of these behaviors can be materialized by the following description of the leader competency within the graduate model of the Air Force Academy:

- Describes the procedures and fundamental values of the Air Force regarding ethical management;
- Explains the value of membership and the roles played by leader and follower within an organization. Adopts the values and standards of the organization, the individual responsibilities as a follower and the role of the individual within it;
- Maintains military bearing or professional etiquette at all times. Places the well-being of colleagues or subordinates before personal needs;
- Explains the importance of lifelong learning, self-assessment, and seeking or incorporating feedback;
- Communicates and formulates the message in a concise, succinct and clear manner to ensure effective communication;
- Listen, paying full attention to the speaker;
- Describe personal contributions and differentiate others' contributions to the effectiveness of the group. Demonstrates the responsibility to help and motivate others to improve their skills and increase their performance, through personal example;
- Explains and demonstrates the concept of teammate. Illustrates the binder that connects all air force members and reflects full security, trust and mutual support both during and outside service;
- Understands the importance of diversity, including mutual respect, thus contributing to the promotion and strengthening of a culture of air forces that values the inclusion of all personnel within the organization. It supports diversity within the Romanian Air Force;
- Define the mission, specify its expectations and direct the team;
- Describes the full spectrum of military operations identified in the concepts of military theory, air force doctrine, and national defense strategy;
- Acknowledges how technology enhances the operational capabilities of air forces, the ability to achieve interoperability with other joint, coalition, interdepartmental and non-governmental organizations;
- Identifies the structural elements of the air forces and the joint and interdepartmental relations; describes how its function and unit fit into the organization and how it relates to the external environment - organizations it supports or is supported by;
- Identifies the factors that influence the defense policy, internal and external; try to understand the cultural and linguistic norms and customs in the areas of interest and is able to recognize the differences between cultures at the regional level;
- Identifies, acquires and conserves the resources (financial, informational, technological, material, energy and human) necessary to accomplish the mission. Follow the deadlines and stages set for the execution of the mission. Uses the resources, as indicated or available;

- It describes the importance of proactively adopting, supporting and leading the process of continuous change and improvement of the air forces. Adopts and supports organizational changes in air forces;
- Identifies and seeks opportunities to improve existing processes and conditions;
- Accepts the value that the vision has in fulfilling the mission and choosing the right paths for the efficient management of the teams and the achievement of the objectives;
- Identifies, evaluates and assimilates information, from multiple sources, according to utility. Apply this information to influence actions and decisions;
- Accepts change and maintains efficiency when faced with changes in the work process, demands or new cultures encountered.

In the same direction, a great responsibility rests with the Air Force Application School "Aurel Vlaicu", in the sense of modifying or redesigning the graduate model in order to meet all the wishes expressed by the answers obtained from the beneficiaries of the educational institutions.

The training programs should be structured in such a way that the desired behaviors displayed by the trainee officers rise to the level expressed by the specified model. The responsibility to do this falls on the shoulders of the instructors in this way, by identifying those contents that will help them design those programs.

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A METHOD FOR DETERMINING THE COMBAT POSSIBILITIES OF MILITARY UNITS

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Abstract:*Based on the known factor of military unit combat power, the main interest of this paper is to consier the problem of combat possibilities calculation. By using estimations, the method of tactical operation calculation of the task implementation possibility, the factor of battle parties losses and the attack movement rate are povided.*

Keywords: combat power, task implementation probability, losses, attack rate, interpolation

1. INTRODUCTION

The assessment methods of combat possibilities have a special position among the support systems of the process of military decision-making [1]. The correct decision-making as far as combat activities are concerned and the impartial assessment of combat possibilities of the battle both sides for define of combat tasks have much significance. For lack of reliable methods in this area the commanders are reluctant to make intuitive decision based only on the own practical experience.

The assessment of combat possibilities is not new problem [2] and in this area many investigations are carried out [3,4]. However, for practical calculation of combat possibilities in the various offered methods the fire, blow and maneuver possibilities are defined on the basis of various methods, and the quantity describing the common combat possibility of unit is not calculated. It complicates to implement of tactical operational calculations on its basis.

In the paper, on the basis of dynamic power of the military unit [5] described its combat possibilities the tactical operational calculations implementation method has been offered.

2. CALCULATION OF ELATIVELY COMBAT POWER AND PROBABILITY OF TASKS IMPLEMENTATION

For the purpose of assessment of expected battle victory we can calculate a combat power ratio of battle both sides (both for attack and defence sides) [5]:

$$GN_h = \frac{G_D^h}{G_D^m};\ GN_m = \frac{G_D^m}{G_D^h} \qquad (1)$$

Here: GN_h – is a combat power ratio for the attack side, G_D^h – is a combat dynamic power for the attack side, GN_m – is a combat power ratio for the defence side, G_D^m – is a combat dynamic power for the defence side. If GN_h >1 then it is means that a attack side will has a reliable victory and the propability of its success P_T is directly proportional to value of GN_h [5].

During attack some of time after beginning, the content of attacking troop for utter defeat of the enemy is defined by the level of possible losses of the sides. The dependence between the combat power ratio of sides (*GN*) and the possible losses is determined by below formula [5]:

$$\dot{I}_h = 1 - \sqrt{1 - GN_m^2 \times \dot{I}_m \times (2 - \dot{I}_m)} \quad (2)$$

$$\dot{I}_m = 1 - \sqrt{1 - GN_h^2 \times \dot{I}_h \times (2 - \dot{I}_h)} \quad (3)$$

Here: $\dot{I}_h$ – is a losses factor of attacking side, $\dot{I}_m$ – is a losses factor of defending. Its quantities change between (0÷1) range.

The wars' expirience has shown if an attacking side has 30-50% losess ($\dot{I}_h = 0{,}3 \div 0{,}5$ then it gives up the attack or it must organize a new attack. If an attacking side has more 50% losess, in this case the continuation of attack is not possible. If a defending side has 50-70% losess then its combat possibilities decrease much, and it must organize a defence over again. If a defending side has more 70% losess, in this case it cannot defend itself at all [5].

If the losesses for an attacking side are 30-50% and for a defending side are 50-70% the these losesses are called critical. From this we can calculate a combat critical power ration for battle both sides. If $GN_h = GN_{h.crit\,ic}$ and $GN_m = GN_{m.critic}$, then after battle beginning, at the same time, both sides losess are begun critical, and in this case the combat tasks implementation possiilities for both sides are equal. If, $GN_h < GN_{h.critic}$ and $GN_m > GN_{m.critic}$, then a defending side will obtain rather critical losess and for an attacking side the combat tasks implementation possiilities will more much. Below formulas can be used for calculation of both sides critical ratio [5]:

$$GN_{h,critic} = \sqrt{\frac{(2\times\dot{I}_m - \dot{I}_m^2)}{(2\times\dot{I}_h - \dot{I}_h^2)}} \quad (4)$$

$$GN_{m,critic} = \sqrt{\frac{(2\times\dot{I}_h - \dot{I}_h^2)}{(2\times\dot{I}_m - \dot{I}_m^2)}} \quad (5)$$

There have been shown below in table 1 the calculated values of critical ratio of the combat power for both attacking ($GN_{h.critic}$ in denominator) and defending sides ($GN_{m.critic}$ in denominator).

Table 1. *The critical ratio of the combat power for both sides (GN).*

		Losess of attacking side				
		30%	35%	40%	45%	50%
The losess of defending side	50%	1.213 0.821	1.140 0.878	1.083 0.924	1.037 0.964	1.000 1.000
	55%	1.251 0.800	1.175 0.851	1.116 0.900	1.069 0.935	1.031 0.970
	60%	1.283 0.779	1.206 0.829	1.146 0.873	1.097 0.913	1.058 0.945
	65%	1.312 0.762	1.233 0.811	1.171 0.854	1.122 0.892	1.082 0.925
	70%	1.336 0.749	1.255 0.797	1.192 0.839	1.142 0.876	1.102 0.908

For the pupose of break open and exploit an attack deep in the enemy defense for the retention of combat possibilities the attacking side losess must be less 30% ($\dot{I}_h \leq 0.3$.

In this case, the defending side will has the minimum 70% losess ($\dot{I}_m \geq 0.7$ and will crushed. For such losess the appropriate relative combat power of attaking side is called full one (a full superiority) [5]. By using formula (4) let us calculate its value:

$$GN_{h.full} = \sqrt{\frac{2 \times 0.7 - 0.7^2}{2 \times 0.3 - 0.3^2}} = 1.34$$

Therefore, if $GN_h \geq GN_{h.full}$ then the attacking side has much probability to implement combat task. By analogy, the defending side losess must be less 50% ($\dot{I}_m \leq 0.5$ that to provide defence capability. In this case the attacking side must to has more 50% losess ($\dot{I}_h \geq 0.5$. For such losess the appropriate relative combat power of defending side is called full one (a full superiority). By using formula (5) let us calculate its value:

$$GN_{m.full} = \sqrt{\frac{2 \times 0.5 - 0.5^2}{2 \times 0.5 - 0.5^2}} = 1$$

Therefore, if $GN_m \geq GN_{m.full}$ then the defending side has much probability to implement combat task.

In the dependence of combat task implementation probability on some values of relatively combat power and the method of probability calculation have been presented [6]. However, thre are the inverse tasks in many practical problems: for given probability of task implementaton the necessary relatively combat power should be determine. For this porpuse let us determine $P_T = F_P(GN| \quad |h)$ and $GN_h = F_G(P| \quad |T)$functions by using of coefficients given in table 2.

Table 2. *The probability of task implementaton and the dependence of relatively combat power.*

Given $F_P(GN| \quad |h)$	GN_h	2.18	1.81	1.59	1.44	1.34	1.26	1
	P_T	0.71	0.65	0.6	0.56	0.54	0.51	0.5
Calculated by interpolation $F_P(GN| \quad |h)$	P_T	0.71	0.65	0.60	0.56	0.54	0.52	0.50
	Error	0.00	0.00	0.00	0.00	0.00	0.01	0.00
Calculated by interpolation $F_G(P| \quad |T)$	GN_h	2.18	1.76	1.59	1.45	1.34	1.10	1.00
	Error	0.00	0.05	0.00	0.01	0.00	0.16	0.00

For determination these functions let us use Newton's interpolation formula:

$$P_n(x) = a_0 + a_1 \times (x - x_0) + a_2 \times (x - x_0) \times (x - x_1) + \dots + a_n \times (x - x_0) \times (x - x_1) \times \dots \times (x - x_n)a_0 + \sum_{i=1,n} a_i \times \prod_{j=0,i-1}(x - x_i) \tag{6}$$

a_0 is obtained from $P_n(x_i) = y_i$. If *i=0*, $a_0 = y_0$; *i=1* then $a_0 + a_1 \times (x_1 - x_0) = y_0$, From here $a_1 = (y| \quad |1 - y_0)/(x_1 - x_0)$.

By analogy, the another factors can be calculated.

For interpolation by using data from table 2 (1, 3, 5, 7 columns) below factors have been obtained:

– for $F_P(GN| \quad |h)$ function: $a_0 = 1.15; a_1 = -1.53; a_2 = 1.11; a_3 = -0.23$
– for $F_G(P| \quad |T)$ function:$a_0 = 53.81, a_1 = 268.02; a_2 = -436{,}73; a_3 = 239.88$

From interpolation formula obtained $F_P(GN| \quad |h)$ and $F_G(P| \quad |T)$ values are given in table 1. The low errors allow to assert that factors are quite adequate. Let us write these dependences as:

$$P_T = 1.15 - 1.53 \times GN_h + 1.11 \times GN_h^2 - 0.23 \times GN_h^3 \tag{7}$$

$$GN_h = -53.81 + 268.02 \times P_T - 436{,}73 \times P_T^2 + 239.88 \times P_T^3 \tag{8}$$

During headquarter activities the operation-tactic target setting with necessary calculations can be various. However, there are three main parts for each tasks: the content of attacking side and its dynamic power G_D^h; the content of defending side and its dynamic power G_D^m; combat task and its implementation probability P_T. From this point of view there is a below classification of tasks:

I-st type task - G_D^h and G_D^m are given, P_T is found;

II-nd type task - G_D^h and P_T are given, G_D^m is found;

III-rd type task - G_D^m and P_T are given, G_D^h is found.

In I-st type task (7) formula can be applied for determination of probability of task implementation for both attacking and defending sides:

$$P_T^h = 1.1498 - 1.1498 \times \frac{G_D^h}{G_D^m} + 1.1104 \times \tag{9}$$

$$P_T^m = 1.1498 - 1.1498 \times \frac{G_D^m}{G_D^h} + 1.1104 \times \tag{10}$$

II-nd and III-rd type tasks have been solved usually combat task setting during atack (defence) planning. In this time if the unit's content and task implementation probability are known then (8) formula can be applied:

$$G_D^m = G_D^h / \tag{11}$$

$$G_D^h = G_D^m \times \tag{12}$$

3. CALCULATION OF LOSSES

For the purpose of losses calculation by using (1) and (2) equations the losses of one side must been known. However, in some time the both sides' losses must been known. For this purpose to combat power ratio of both attacking and defending sides appropriate the values based on the practical losses expirience can used (K.K.K.: YY-8: 1986). Let us determine a functional relationship between losses and relatively combat power by using of Newton's interpolation formula. For interpolation below factors have been obtained:

- in attack: a_0 = 2.4179; a_1 = -1.8032;
  a_2 = 0.5395; a_3 = -0.0524
- in defence: a_0 = 0.3086; a_1 = 0.0884;
  a_2 = 0.0417; a_3 = 0.0016.

Functional relationships between losses and relatively combat power for unit during 1 hour battle have been expressed as:

$$\dot{I}_h = 2.4179 - 1.8032 \times GN_h + 0.5395 \times GN_h^2 - 0.0524 \times GN_h^3 \tag{13}$$

$$\dot{I}_m = 0.3086 + 0.0884 \times GN_h + 0.0417 \times GN_h^2 + 0.0016 \times GN_h^3 \tag{14}$$

Calculated by interpolation equation and shown in figures 1 and 2 the dependnces between losses and relatively combat power allow to conclude that the results of interpolation are quite adequate.

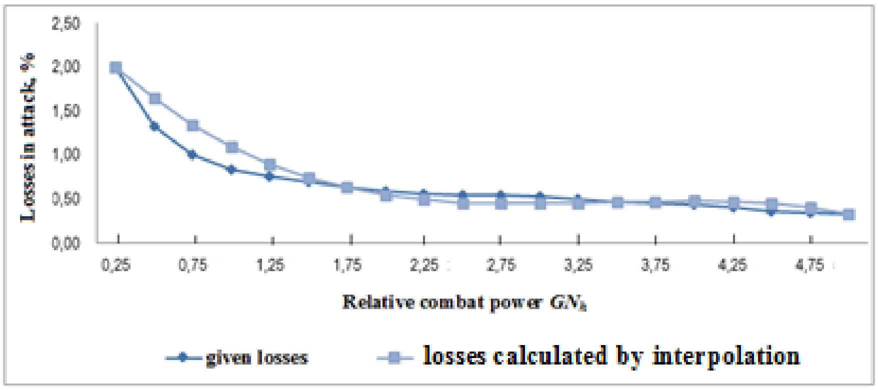

FIG.1 The dependence of losses on relatively combat power in attack.

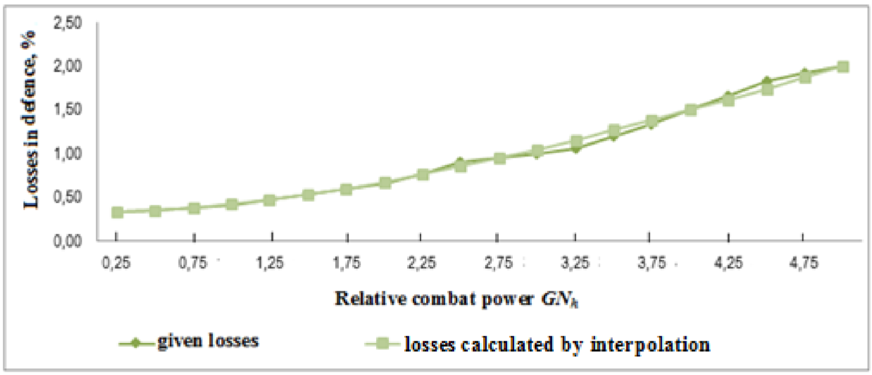

FIG.2 *The dependence of losses on relatively combat power in defence*

When the unit's losses have been determined then its should be divided on the categories of personal, weapons and military technics. İn this case the losses for various categories will different. It is conditioned by application properties on the battle area. The personal, weapons and military technics losses's factors given in [7]. The losses of unit's weapons and military technics can calculated by below equation:

$$\dot{I}SH_j = \dot{I} \times N_j \times K_j, j = I, II, \ldots IX \tag{15}$$

Here: $\dot{I}SH_j$ is a number of j weapon category losses, $\dot{I}$ is the factor of unit's common losses, N_j is a number of weapons of j category, K_j is a losses factor of j weapon category. For calculation of unit's personal losses balow equation is used:

$$\dot{I}ŞH = \sum_{j=I,\ldots,IX} \dot{I}SH_j \times Ş_j \tag{16}$$

Here: $\dot{I}SH_j$ is losses for each weapon's category; $Ş_j$ is a factor of personal losses for the same weapon's category.

4. CALCULATION OF ATTACK RATE

The unit's attack rate is depended on many factors: the type of unit, the conditions of terrain and weather, the disposition of defence and the engineering fortifications.

The units of measurement of the attack rate are km/hour for (squad/ brigade) and km/day for (army's corps/ army). Here, km/day cannot displace to km/hour, because attack rate are not same in day and night, it is taken as average quantity.

First of all, let us consider attack rate during 1 hour. The war and military trainings expirience have shown that in dependence on the ratio of both sides combat power the attacking side rate is given in [7]. For the pupose of determination of the functional dependence between ratio of combat power and attack rate let us use Newton's interpolation formula (6) and determine a_i factors. Taking into account of directly proportional smooth dependence between unit's attack rate and relatively combat power let us take this dependence as non-linear one of third degree. For interpolation below factors have been obtained: a_o = -1.0631; a_1 = 1.4952; a_2 = -0.4900; a_3=0.0579. The dependence of unit's progress common rate (km/hour) in attack on relatively combat power can calculated by below equation:

$$CHT_{hour} = -1.0631 + 1.4952 \times GN_h - 0.4900 \times GN_h^2 + 0.0579 \times GN_h^3 \qquad (17)$$

The dependence of progres rate in km/hour on combat power (figure 3) shows that interpolation results are adequate.

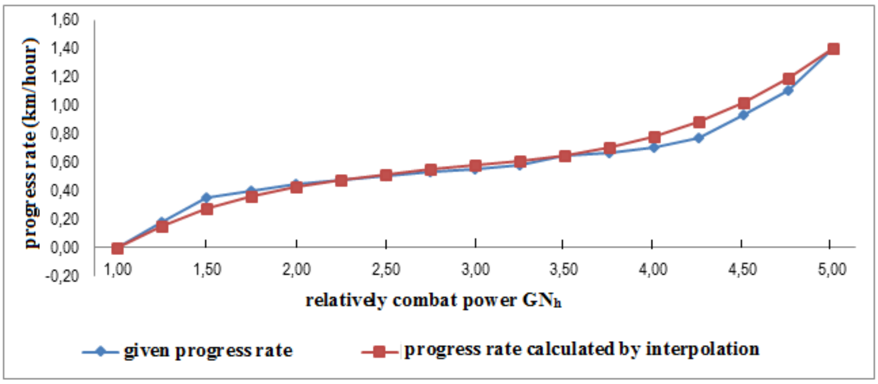

FIG. 3 The dependence of progress common rate in attack on relatively combat power

Now, let us consider a progress rate during 1 day for brigade and bigger military formations in attack operations. Taking into account that in attacking operations during all day long the progress rate are different then 24xCHT_{hour} / CHT_{day}. The losses data during both 1 hour and 1 day have been taken from different sources [7,8]. Let us determine the functional dependence between relatively combat power and losses. By using of Newton's interpolation formula below interpolation factors have been obtained: a_0=1.4711; a_1=1.6067; a_2=0.8575; a_3 = -0.0895.

In attack the dependence of unit's progress common rate (in km/day) on relatively combat power can expressed below equation:

$$CHT_{day} = 1.4711 + 1.6067 \times GN_h + 0.8575 \times GN_h^2 - 0.0895 \times GN_h^3 \qquad (18)$$

The result of interpolation of the attacking progress rate in km/day are shown in figure 4.

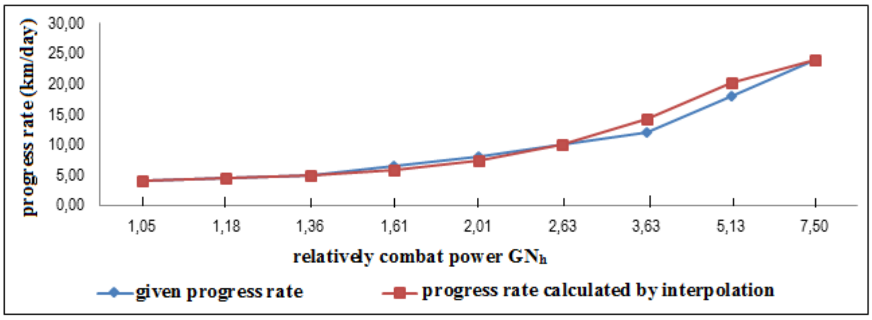

FIG. 4 The dependence of attacking progress rate on relatively combat power.

In attack the unit's really progress rate is depended on the type of unit, terrain and weather conditions, time of day (if rate is measered in km/hour) and the type of fortifications. These factors must be considered for calculation of attacking progress rate. These factors are given in [7,8]. Taking into account of these factors unit's attacking progress rate is expresses by next equation:

$$HT_{day} = CHT_{day} \times WC \times TC \times BT \times MS \times SV \tag{19}$$

$$HT_{day} = CHT_{day} \times WC \times TC \times BT \times MS \tag{20}$$

Here: *HT* is an attacking progress rate; *CHT* is a attacking progress common rate (9, 10); *WC* is weather influence factor; *TC* is terrain influence factor; *BT* is unit's type influence factor; *MS* is fortifications' density influence factor; *SV* is time of day influence factor.

During planning of military operations (tactic activities) the inverse problem can be often considered: the Staff determines an attacking progress rate and the unit's commander must determine necessary power ratio. Thereto, let us determine the dependence of combat power ratio on attacking rate. By using of Newton's interpolation formula below functional dependence's factors have been obtained:

a_0 = -0.3424; a_1=0.3940; a_2 = -0.0131; a_3=0.0003.

By using of these factors the below functional dependence of attacking side's combat power ratio on the given attacking rate:

$$GN_h = -0.3424 + 0{,}3940 \times CHT_{day} - 0.0131 \times CHT_{day}^2 + 0.0003 \times CHT_{day}^3 \tag{21}$$

Taking into account of (21) equation the next formula is obtained:

$$GN_h = -0.3424 + 0{,}3940 \times \frac{HT_{day}}{WC \times TC \times BT \times MS} - 0.0131 \times \left(\frac{HT_{day}}{WC \times TC \times BT \times MS}\right)^2 + 0.0003 \times \left(\frac{HT_{day}}{WC \times TC \times BT \times MS}\right)^3$$

CONCLUSION

Therefore, the method of tactical operation calculation of the task implementation possibility has been developed and offered. On the basis of the unit's dynamic combat power the some factors defined combat capability are determined. There are relatively combat power, including critical and full, combat task implementation probability, both sides losses and attacking progress rate among these factors. The equations for calculation these factors have been obtainned.

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AN ANALYSIS OF THE CYBER DIMENSION IN HYBRID OPERATIONS

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Abstract: *The accelerated technological advances in the field of communications and information systems have produced profound changes in society. Over time, these changes have become prerequisites for good functioning. Moving the area of interest into the virtual space generated by cyber infrastructures represented a migration of threats. Thus, the possibility of remote control of cyber infrastructures, both in peacetime and in conflict, has generated a new type of infrastructure for the national defense system – the critical military cyber infrastructure. Significant contributions are made to the proactive management of critical infrastructures by developing a multi-level architecture model of active cyber defense in the context of hybrid threats.*

Keywords: *hybrid operations, active cyber defense, military critical cyber infrastructures, antifragile*

1. INTRODUCTION

The integration of information and communication technologies (ICTs) in all organizational levels, as an essential condition for progress, has taken the form of "technology push" programs in the military operational field in order to remove the fear of being "left behind".

The notion of cyberspace is not new (Gibsonian Cyberspace) [1], but further developments in the field of ICTs have determined its definition in terms of transversality in relation to the aerial, ground, maritime and space operational environments.

If the traditional military dimension of the national security is national defense, cyber security is an important support for ensuring national security by interconnecting all its domains: national security, critical infrastructure protection, civil protection, public and constitutional order [2].

In this context, cyber defense consists of the set of proactive and reactive measures, military and civilian, which contribute to maintaining the state of normality in the cyber space [3]. The state of normality is disturbed when cyber threats are manifested. Thus, cyber-attacks are persistently initiated throughout the duration of the political-military crises, but also during peace time.

The favorite target of cyber-attacks is represented by the critical assets in the field of ICT (e.g. computer systems, networks, computer programs, electronic communications networks), also known as the critical cyber infrastructure. Criticality is associated with an element or network of elements essential to maintain vital societal functions [4].

The complexity and diversity of both critical cyber infrastructures and cyber threats, involves the development and application of specific complementary protection and resilience measures [5], integrated with the risk management process. The resilience of critical cyber infrastructures, in terms of maintaining the functionality when producing shocks and adapting to changes in the action environment, is possible by following the next principles of risk management [6]:

- the risk cannot be completely eliminated;
- the nature of risk perceptions and behavioral biases should not be ignored;
- a diversified portfolio of measures contributes to efficient risk management and resource efficiency;
- threat identification and risk assessments represent a critical input to the decision-making process;
- risk communication is a critical aspect at organizational level.

Estimating the risk of a cyber-attack on a critical infrastructure is a continuous and complex process capable to identify possible threats, their evolution in terms of probability of manifestation and possible consequences, vulnerabilities and measures to counteract the effects (FIG. 1).

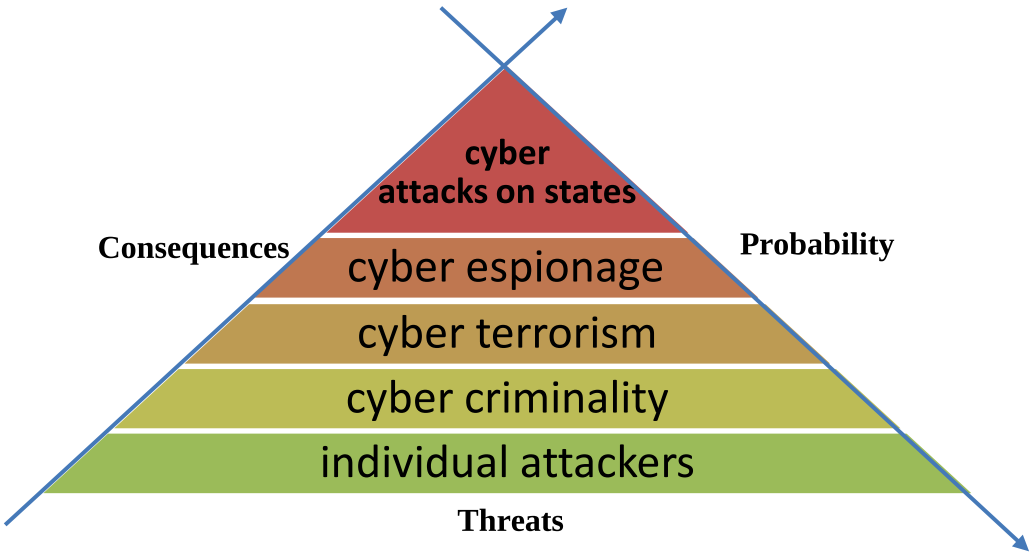

FIG. 1 Elements of risk analysis

A cyberattack on a state can be carried out in peacetime (e.g. Estonia, 2007), before the escalation of a crisis (e.g. Ukraine, 2014) or simultaneously with the military aggression (e.g. Georgia, 2008). Thus, the delimitation between cybercrime actions and cyber attacks as acts of war is difficult and depends more on the context (the crisis' position on the evolution curve).

The risks in the cyber space proliferate due to the interconnected and dependent infrastructures. The management tools and responsibilities must be designed in an integrated manner.

There are enough arguments to declare that cyber security has become the biggest concern of security structures/ organizations, overcoming terrorism. The analysis of the institutional responsibilities in the field of cyber security emphasizes the two-dimensional integrated approach by purpose: good governance (during peace/ stability time) and cyber defense (in crisis situations). Thus, Table 1 shows the main institutions and associated responsibilities at national level from the perspective of good governance [6].

Table 1. The national cyber security system

Institutions	Responsibilities	Priorities 2018
Cyber Security Operative Council (COSC)	-coordinating actions at national level to ensure cyber space security	- ensuring inter-institutional cooperation; - intensifying cooperation actions with international partners
National Center for Response to Cyber Security Incidents (CERT-RO)	- develops and disseminates public policies to prevent and counteract cyber incidents	- the development of technical and human capabilities; - adopting the institutional framework to the new requirements imposed by transposition into national law
Cyberint National Center (CNC)	- managing information capacities to provide beneficiaries with the information they need to prevent, limit and/ or stem the consequences of cyber aggression on national critical infrastructures	- Internet control for the knowledge, prevention and counteraction of Romania's vulnerabilities, risks and threats to cyber security

The Supreme Council of National Defense (CSAT) is the institution that coordinates the activity of the National Cyber Security System (SNSC), and within it, the institutions represented in the COSC develop an intense cooperation with the international institutions in the field of competence [3].

From the operational perspective, cyber defense is the attribute of the Ministry of National Defense, which: defends the cyber systems and infrastructures belonging to the Ministry of National Defense (through the National Technical Center for Cyber Security Incident Response CERTMIL-CTP); plans, conducts and executes operations in the cyber space (through the Defense Staff); ensures cooperation and exchange of information with NATO military entities [7].

At the allied level, the new concepts and policies in the field of cyber defense initiated at the 2014 Summit were perfected in Warsaw in 2016, when the cyber space was recognized as an operational area, as a part of the collective defense, also in the hybrid context. Thus, the cyber defense began to be integrated in the operations planning at all levels (new Cyber Operations Center operational from 2023), the use of the cyber capabilities of the alliance being realized in compliance with the provisions of international law. [8]

NATO Computer Incident Response Capability (NCIRC) is the specialized structure to continuously protect the networks used by NATO. In order to develop and maintain capabilities at national level, cyber defense has also been integrated into the Smart Defense projects (MISP, Smart Defense Multinational Cyber Defense Capability Development - MNCD2, Multinational Cyber Defense Education and Training - MNCDE & T).

Immediate priorities for the alliance are: strengthening cyber security of national infrastructures and networks (Cyber Defense Pledge), as well as enhancing complementary inter-institutional collaborations in the sense of integrated cyber security (by avoiding unnecessary duplication of effort). NATO-EU collaboration is particularly important in this area in order to achieve the "fit for the future" goal. Thus, the foundation of the Center of Excellence for Hybrid Threats, the signing of the EU-NATO Joint Declaration of Cooperation and the signing of the NATO-EU Technical Cooperation Agreement, are three initiatives aimed at finding solutions, exchanging information and good practices, and at coordinating the actions meant to ensure the cyber defense.

At European level, with the adoption of the security and defense plan (2016), the foundations of permanent structured cooperation (PESCO) were laid, and a first set of 17 collaboration projects that respond to training, development capabilities and operational availability needs have been initiated since 2018.

Romania has established responsibilities for 5 of the 17 projects, one of them being "Teams for rapid response and mutual assistance in the field of cyber security", a project coordinated by Lithuania.

In the case of good governance, as well as for cyber defense, managing these dynamic cross-border and trans-organizational threats involves: trained human resources; a consistent approach to cyber defense capabilities in an allied environment; coordinated response actions.

2. THE CYBER DIMENSION OF HYBRID ACTIONS

At the beginning of the 21st century, the use of the term "hybrid" became a common way to describe the contemporary war, at least from two arguments: the increasingly important role of non-state actors in the dynamics of the security environment and the escalation of cyber operations.

Debates over the concept of *hybrid warfare* have shaped at least two sides: on the one hand, there are specialists who consider hybrid warfare a reality that needs its own approach, and on the other, those who claim that hybrid warfare just defines something that has existed throughout the history of the war.

Thus, it is not surprising that there are many definitions of hybrid warfare. The concept has been shaped in different ways, and these definitions have evolved in a relatively short period of time. Defining hybrid warfare is not just an academic exercise. The way the concept is defined leads to the outline of threat perception and the proactive and reactive manner of action. For this study, we considered three conceptual approaches to hybrid warfare, whose chronology fit the Crimea 2014 moment (Tab. 2).

Less than 7 years after the onset of the crisis in Ukraine, a conceptual approach of the two approaches is outlined, in the sense that:

- the traditional and irregular war is sufficient to describe the current and future operational environment;

- each conflict has its own particularities (in terms of the methods used to exploit the adversaries' vulnerabilities).

Table 2. Definitions of hybrid war

Definition	Author
Threats that incorporate different combat modes, including conventional capabilities, irregular tactics and formations, terrorist acts including violence and coercion, driven by a variety of non-state actors.	Hoffman, 2007 [9]
Using military forces in an auxiliary way to non-military tactics to achieve strategic and political goals amid the creation and exploitation of an environment of worry and permanent conflict.	Gerasimov, 2013 [10]
Using military and non-military instruments in an integrated campaign, designed to surprise, take advantage of the initiative.	The Military Balance, 2015 [11]

The inductive analysis of the operations that Russia has carried out in Crimea and Eastern Ukraine outlines four main stages of the crisis, each of them divided into sections, but without considering a linear development (FIG. 2).

According to some experts, from a chronological point of view, the hybrid operations carried out during the crisis in Ukraine covered only the escalation phase of the crisis, from the second half of February to the second half of May 2014 [12].

According to the presented definitions, the synchronized and coordinated use of power tools (military, political, economic, civil and informational), alternatively or coupled, with varying intensities, was encountered throughout the crisis [13].

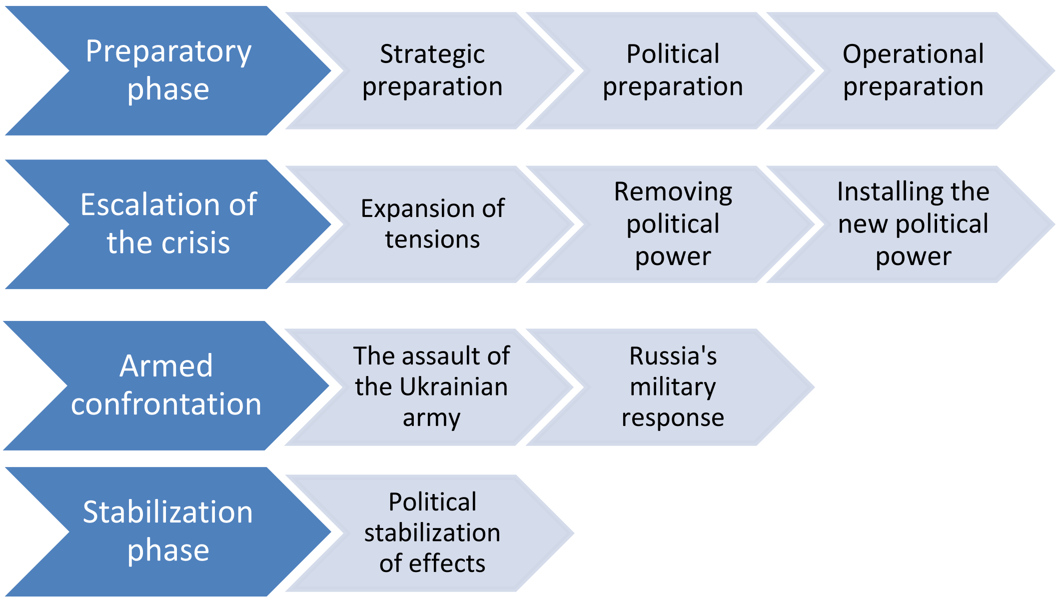

FIG. 2 The stages of the Ukraine crisis

Although it had been successfully tested in the Russian-Georgian conflict of 2008 (when it was used at the same time as the start of military operations), the cyber weapon became the main component of the information instrument in the hybrid war. In Ukraine, cyber-attacks (hacking, denial of service, boot, civilian trolls) were launched before the armed confrontation on both government institutions and the army, with the aim of creating dissensions in society, confusion and imbalance [14].

Russia understood the strategic importance of the cyber space and exploited it beyond the traditional operational capabilities that define the Euro-Atlantic political and military response mechanisms [15]. From the moment of Georgia 2008 to the moment of Ukraine 2014, there is an adaptation of the attack strategies and the targets to the specific objectives (Tab. 3).

Table 3. Features of cyber attacks in Georgia and Ukraine

Attack detail	Georgia	Ukraine
Period	August 2008	February - May 2014
Type of attack	DDoS	DDoS Wipper Bot Physical
Targets	Government agencies Media organizations	government and military websites
Scope	Stopping of communications	Isolation of the region and creation of premises for military operations
Complexity	Simple/ disorganized	Sophisticated/ organized
Context	At the same time with the military operations	before the Crimean invasion and supporting separatists in Eastern Ukraine

Cyber capabilities, within hybrid operations, represent tools at the border between hard power and soft power, with a pronounced offensive character [15].

The cyber weapon can be used not only against institutions with responsibilities in the field of national security (Ministry of National Defense, Ministry of Internal Affairs, Romanian Intelligence Service), but also against other key sectors of the society: financial market, media organizations, science and research, education, health, civil society.

This threat easily crosses the sectorial boundaries in a sequential or simultaneous manner, which leads to a comprehensive approach, based on: common understanding and awareness of the situation; defense planning; efficient leverage of the company's resources; sectorial, national and regional partnerships; lessons learned and best practices.

The inclusion of military objectives/ infrastructure on the target list of aggressive cyber operations by a state using cyber capabilities is the main argument for using the *cyber warfare* concept [16].

3. MODEL OF ACTIVE CYBER DEFENSE ARCHITECTURE

Active cyber defense is a new concept that facilitates the effort unit by integrating, synchronizing and automating cyber defense capabilities across all government networks and critical infrastructure in the US [17]. For this scientific research, in defining the model of active cyber defense in hybrid operations, we used the gradual properties of the cyber systems associated with the three dimensions of the cyber space: technological, informational and socio-cultural (Fig. 3).

The three dimensions of cyber space must be interpreted as a fusion of information systems, the Internet and people in order to create a global virtual domain that provides the premises for competitive advantages [18].

In each dimension, the presence of potential vulnerability factors in terms of cyber security is noted, such as: oversizing investments in knowledge of threats, as compared to investments in protection measures; flexibility in the implementation of standards and sub-optimal use of resources; inadequate human-machine communication; resistance to change.

Resistance is the level of protection against a certain type of threat, being a specific property. Once this capability is developed, the system will avoid the change and risks associated with it, continuing to operate in the same architecture. Resistance also includes elements of redundancy, pending or used concurrently, in order to absorb shocks [19].

Resilience is the ability to adapt in response to the danger of a cyber-attack that allows the system to avoid some potential losses. For this scientific approach, resilient systems contain the following combination of qualities: flexibility, adaptability, inclusivity and integration.

Antifragility, according to Taleb's theory (2014), captures the positive impact of shocks on the system after it has become resilient (adaptable to changes in the operational environment) [20]. Antifragile systems, without the ability to learn from incidents, become fragile over time due to the changes that occur both inside and in the environment in which they operate [21]. Thus, by activating antifragility in the cyber domain, a better understanding of the sectoral interrelationships and the premises for a functional security is ensured.

The properties of active cyber defense can be interpreted in terms of successive levels of capacity (Fig. 4). On the first level, resistance, there are passive defense mechanisms that serve to strengthen and fortify the critical infrastructure. At the next level there are the defense capabilities resulting from the use of defensive cyber weapons and dual-use weapons: communications camouflage, content camouflage, disaster recovery systems. They give the network resilience through adaptation, thus ensuring the continuation of operations despite ongoing attacks.

At the last level, the technical operations of cyber defense are innovative: AI, machine learning, threat intelligence. The resistance and resilience of critical cyber infrastructures ensure their survival by acquiring capabilities with a defensive profile that will lead to reducing the chances of success of a malicious attack.

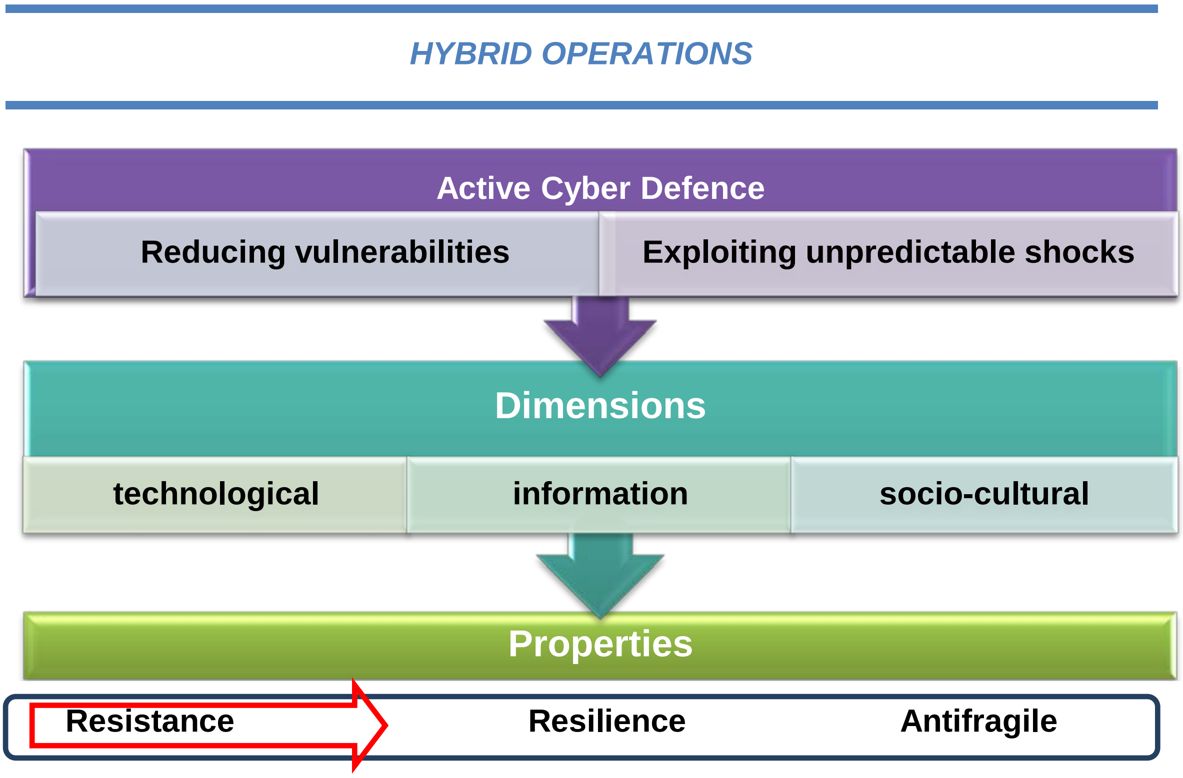

FIG. 3 The architecture of the active cyber defense model

This is achieved technically with the help of: firewall, network monitoring, vulnerability scanning, penetrability testing, encryption, content camouflage, communications camouflage, disaster recovery systems.

These technical measures, though effective against a large register of attacks, are ultimately defeated by innovative exploitation of vulnerabilities, including human factor. The security culture at the individual level represents a combination of experience, knowledge, values and security procedures. The vulnerability at the human factor level is not in the local security environment (eg the government institution), but rather in its manifestation in the virtual social environment, where the constraints are lower.

Therefore, through a strategy of optimal use of resources and unitary implementation of standards, operational efficiency can be ensured. Under the conditions of hybrid operations, the challenge related to the share of cyber defense expenditures, compared to those for high-performance weapon systems, out of the total defense expenditures, is perpetuated.

For the first two levels of active cyber defense, it is essential to develop a model of continuity of services, even in breakdown mode, and the rapid return after attack (eg start redundant systems, identification and blocking of the attack vector). It is also important to establish a framework for identifying vulnerable critical nodes, where enhanced resistance and resilience capabilities are implemented. The dynamic modification of the network configuration through modularity, redundancy and diversity reduces the chances of success of the attack.

The level of antifragility is based on the following pillars: awareness of the benefits obtained by applying the theory of optionality for investments in volatile environments; weak interconnections between nodes to prevent attack propagation [21]; developing technical capabilities for feedback, memory and learning; cyber education and research.

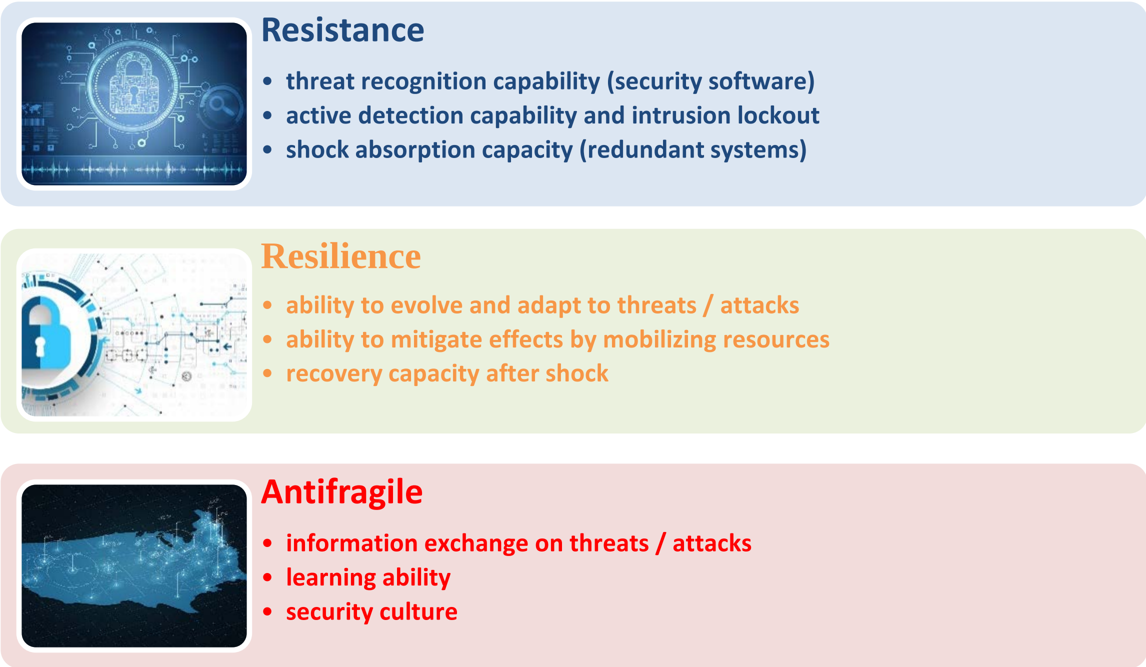

FIG. 4 Multi-level capabilities of active cyber defense

Two main arguments have been identified for defining this architecture:

1. the massive expenses in ensuring cyber security led to ensuring the resistance and resilience of the systems, which proved to be fragile;

2. determining the costs associated with the effects of cyber attacks is a particularly difficult task (includes unknown variables or difficult to estimate), without being able to capture the context [22].

Thus, investments to reduce the risk of survival of critical cyber infrastructures are justified, and what must be changed is the share of these investments in relation to investments in innovation. These investments, although made in volatile conditions, create the premises for small shocks benefits/ gains by incorporating learning from continuous change.

It remains valid the hypothesis developed within the concept of interdependent security [23], which shows the contagion of proximity of security (in an allied context, the level of vulnerability of a national network depends on the level of vulnerability of the other members, and at national level, it depends on the level of vulnerability of the different sectors of the critical infrastructure).

CONCLUSIONS

The guarantee of peace suggested by Harari (2018) through the "lost art of winning wars" [24], does not seem as convincing in the case of hybrid operations: the Russian Federation has felt the "taste of victory" and is expected to have a tailor-made behavior.

In critical cyber and cyber infrastructures (which are complex adaptive systems), according to the theory of shocks, the following logical reasoning is outlined: information is transmitted from the system to the component elements through stress factors, volatility means information, and security cannot exist without volatility. The investment projects for active cyber defense of critical infrastructures under the conditions of hybrid actions prove a great potential for bidirectional multiplication of the investment: in financial terms and operationally (performance level).

Through such a security approach, a "cyber 9/11" is not possible: even if the effects are significant, they will not be able to reach the catastrophic level. The ability to gain dominance in the cyber space over critical infrastructures is the key to future hybrid actions, and the RESISTANCE - RESILIENCE - ANTIFRAILABILITY architecture will not only ensure their survival, but will make them stronger.

Future research will lead to the introduction of elements that detail the area of innovative technological impact and the ability to learn from continuous change in an analytical model that captures system shocks (cyber attacks that do not affect its survival) with the help of Poisson distribution.

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CYBERWARFARE POLIHEURISTIC APPROACH OF NATION-STATE EXPANSION

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Abstract: *The article aims to provide a way of researching the mechanisms of nation state expansion by using economic means and cyberwarfare. The justification for such a study is dictated by the international dynamics of rewriting the various international agreements between states and their reordering by using means of a hybrid nature that include economic tools, cyberwarfare, mass communication and of course, military techniques. To study these mechanisms of expansion when it comes to the already established superpowers as well as the ascending powers, we rely on the theory of hegemonic war proposed by Robert Gilpin and the foreign policy paradigm issued by Alex Mintz. The specific matrix of decision-making mechanisms will be built on ~~in~~ the evaluation of share rates ~~to~~ via a qualitative-quantitative hybrid approach meant to bring additional value to the market of security studies on ~~of~~ cyberwarfare, which currently encompasses purely rationalist research only. The present approach is not intended as an exhaustive outlook, but rather as a first step towards a larger study, as its purpose is to introduce new and revolutionary ideas in the cyberwarfare research of international relations.*

Keywords: *decision matrix, foreign policy management style, decision rules*

1. INTRODUCTION

The research aims to present a model of the expansion of a nation-state based on the hegemonic theory of Robert Gilpin [1] but adapted to the poliheuristic analysis of cyberwarfare [2]. The theory of hegemonic war by ~~of~~ Robert Gilpin is part of the family of theories of offensive realism and deals with internal and external modalities that make possible the expansion or over-expansion of a state actor depending on economic costs and good governance / internal corruption [1]. In order to develop Gilpin's idea, he considers that the expansion and over-expansion of a state (that is, the economically unsustainable expansion leading to its failure) are determined by three internal economic costs (cost for national security; cost for private and nonmilitary public; cost for productive investment) and the level of corruption or good internal governance. These elements are tight in what we called the "Gilpin Test". All these elements are in agreement with the poliheuristic paradigm that will study the part of cyberwarfare that considers that the foreign policy is dependent on the internal state of the state actor [2] and therefore also on the four variables mentioned above. The research aims to explain cyberwarfare not as a singular element in the arena of international relations but as an intrinsic element of the process of military, economic, cultural, etc. expansion of a state actor.

In the specialized literature, attempts are made to define the new field of human cyber-existence. There are researches that define a new modality of virtual political system [3], trying to adapt the classical paradigms or to propose new paradigms of approach [3]. Most of these studies call for the rational choice paradigm, which has a multitude of limitations in all areas and therefore in cyberwarfare. That is why this study proposes an alternative of bounded rationality based on the poliheuristic paradigm [2]. The limits of the rational paradigms that currently dominate cyberwarfare research are largely explained in a myriad of research [4] that take into account the computer engineering side but neglect the part of prejudice that governs human decision most of the time. That is why the poliheuristic paradigm comes to fill this void.

This research aims to present a way of researching the expansion of a state actor that uses, besides the classical means and cyberwarfare approaches, taking into account also the prejudice system that governs the executive power of a state and which gives it the necessary confidence in the expansionist actions of the policy. external. The article makes an introduction to the poliheuristic paradigm and its elements in the methodology section and in the results section it makes some recommendations that can help the reader to carry out a practical analysis of an expansion of a nation-state based on cyberwarfare. The research will present the way in which using qualitative-quantitative hybrid methods for predicting statistical indicators [5] and the logical relationships between them, the specific decision rates are determined regularly from the decision-making matrix.

2. METHOD

The article makes in the methodology part a presentation of the poliheuristic paradigm together with the elements that characterize it. The reason for which the poliheuristic paradigm was chosen is the widespread idea among researchers in international relations and the analysis of foreign policy that the internal state of the state actor actually determines its behavior and the result of foreign policy [2] and not just the international political system.

Poliheuristic Approach – the poliheuristic paradigm is part of the family of models of foreign policy analysis initiated by the work of Graham Allison, "The Essence of Decision" [6]. This approach was proposed by Israeli political scientist Alex Mintz as a synthesis approach between schools of cognitive psychology and paradigms of rational choice [...], considering that the decision is based on "cognitive short-cuts heuristics." The approach is called poliheuristics because it considers that any decision is a synthesis between several "cognitive short-cuts heuristics". Other defining characteristics of this paradigm are the two-stage decision and the non-compensatory principle of choice between several options. The decision in the first stage is based on bias (ie "heuristics cognitive short-cuts"), and in the second stage on rational approaches. Thus, the decision process through the prejudices of the first stage eliminates a spectrum of opinions that the decision maker would could be considered in the case of a rational approach. That is why the decision-making approach is bounded rationality. The decision analysis tool is the decision-making matrix, based on the principle of the non-compensatory decision. *This principle considers it important for each decision maker to establish a hierarchy of the objective-result dimensions of this matrix, which simplifies the decision-making process* [2]. The most important of the dimensions of the decision matrix is the internal state system, which contains the supreme constraint [2]. The first stage in eliminating unacceptable dimensions, and in the second stage in establishing the decision rules. The research calls on data collected by Council on Foreign Relations in the bank entitled Cyber Operations Tracker which contains sources about motivated or accidental attacks, on which elements of the actor took place respectively his reaction but also to other sources mentioned in the results section.

Cyber Operations Tracker contains data on cyberwarfare incidents from 2005 to date and allows the results section to explain how to calculate the rates in the poliheuristic decision matrix based on statistical indicators such as the intensity of a particular type of cyber attack the economic costs of the "Gilpin Test" of the expansion of a state actor and its indicator of good governance. Regarding the problem of the rates in the decision matrix that explains the hierarchy of actions taken by a state actor in the enlargement process, a variety of evaluation modalities are presented in the specialized literature. The originality of this article is the use of qualitative-quantitative predictive hybrid methods.

The first element is to establish the model of management in foreign policy – in this first stage, the foreign policy management model is identified, focused on one person, group or several decision groups [7]. The next step in the first step of the decision is to establish the model (s) of "cognitive short-cuts heuristics" bias that according to Mintz fall into the following forms [2]: B01. "Focusing on short-term benefits rather than longer-term problems"; B02. "Preference over preference"; B03. "Locking on one alternative"; B04. "Wishful thinking"; B05. "Post-hoc rationalization"; B06. "Relying on the past"; B07. "Focusing on a narrow range of policy options rather than on a wide range of options"; B08. "Groupthink" B09. "Overconfidence; over-estimating one's capabilities and underestimating one's capabilities"; B10. "Ignoring critical information; denial and avoidance"; B11. "Focusing on only part of the decision problem"; B12. "Turf battles leading to suboptimal decisions"; B13. "Lack of tracking and auditing of prior decisions and plans"; B14. "Polyheuristic bias"; B15. "Shooting from the hip"; B16. "Polythink" B17. "Group polarization effect". The second stage of the first step of the two-stage decision process will focus on establishing the critical objective-outcome dimensions of the decision matrix and rejecting the dimensions that offer unacceptable results. Establishing the hierarchy of critical dimensions will be made based on the principle of noncompensatory decision and B01-B17 bias. A decision matrix has a table appearance in which the lines represent the critical dimensions of the objectives respectively of the results $o_1, \dots, o_m$ followed by the decisive actor and the columns are the actions carried out by him $a_1,...,a_m$ $a_1, \dots, a_n$ (see Table 1). According to the principle of non-compensating decision from the analysis of the facts of the state actor there is a hierarchy of critical objectives $o_1,...,o_m$ $o_1, \dots, o_m$ for an action a_i date of the rates values $r_{1i},...,r_{mi}$, practically, there are no two goals with the same importance.

Table 1. Poliheuristic decision matrix to a state decision-maker.

	a_1	a_2	a_3	...	...	a_n	Weights
o_1	r_{11}	r_{12}	r_{13}	...	...	r_{1n}	w_1
o_2	r_{21}	r_{22}	r_{23}	...	...	r_{2n}	w_2
...	...	...	...	...	...	...	...
o_m	r_{m1}	r_{m2}	r_{m3}	...	...	r_{mn}	w_m
Final choice	FC_1	FC_2	FC_3	...	...	FC_n	

Actions / Alternatives - represents the behavior of foreign policy of the state actor that can have a wide spectrum from military, economic, cyberwarfare, radio-electronic or cooperation actions.

Dimensions - after the elements of the first step of the foreign policy process have been established, it is necessary to establish the critical dimensions of the decision matrix.

These may be elements of foreign policy that have a smaller significance than those of internal politics such as the size of the cyber power index [8] or the five-ring model [9] or other models proposed by other authors [2].

Of course, at least one of the dimensions must be linked to the internal political system as considered by Mintz. *Implications* - are explanations specific to each critical dimension produced by a certain action of the poliheuristic decision matrix. *Ratings* - are numerical values assigned to each critical dimension depending on its specific involvement in a particular action. Rate values can range from -10 (very bad) to +10 (very good), critical dimensions have values between 0 and +10. *Weights* - follow the non-compensatory decision principle and the foreign policy management criterion and explain the hierarchy of critical objectives pursued by a political actor. They have values between 0 and +10.

Example: For this study we use the Council on Foreign Relations Cyber Operations Tracker database [10]. It monitors cyber-attacks activities from around the world from 2005 to the day on four dimensions of each state or non-state actor: Civil society, Government, Military, Private sector (economic). Of course, for non-state actors they do not cover all four dimensions. In this study we will consider all four critical dimensions. Cyber actions developed by a state or non-state actor are the following [10]: 1. Distributed Denial of Service - flooding a server with data packets from its clients, so that it can no longer work; 2. Espionage - the ability of an actor to extract useful information from a computer without the owner's approval; 3. Defacement - changing the content of a web content or account from a computer without the consent of its owner; 4. Data Destruction - the ability to use malware to make a malfunctioning computer or destroy the data it contains; 5. Sabotage - the ability to use malware to remove a system from a critical infrastructure controlled by that computer; 6. Doxing - the activity of identifying on the Internet information about a particular individual or collective actor and making it public with evil intentions against it.

Table 2. Example of poliheuristic decision matrix to a state decision-maker based on Cyber Operations Tracker of Council on Foreign Relations

	DDOS	Espionage	Defacement	Data Destruction	Sabotage	Doxing	Weights
Civil society	r_{11}	r_{12}	r_{13}	r_{14}	r_{15}	r_{16}	w_1
Government	r_{21}	r_{22}	r_{23}	r_{24}	r_{25}	r_{26}	w_2
Military	r_{31}	r_{32}	r_{33}	r_{34}	r_{35}	r_{36}	w_3
Private sector	r_{41}	r_{42}	r_{43}	r_{44}	r_{45}	r_{46}	w_4
Final choice	FC_1	FC_2	FC_3	FC_4	FC_5	FC_6	

Determining the decision rules – after establishing the positive and negative rates from the first stage and excluding the non-critical dimensions (those with negative rates), we will move to the second stage in which, based on the rational choice paradigm, the decision rules of the actor will be established. In the following we will present some decision rules [2] that can be based on a decision based on the principle of the non-compensatory decision and the hierarchy of the objective-result critical dimensions. Of these, the most important is as we have outlined above the internal political system.

For the calculation of average weights, the formula below is used:

$$\overline{w_j} = \frac{1}{n} \cdot \sum_{i=1}^{n} r_{ij} \tag{1}$$

After we have established the hierarchy of objectives for each dimension according to biases B01-B17, from the most important to the least important, we will associate the values of the average weights from the highest in value to the lowest in value (to be consistent with the hierarchical order of the objectives resulting from B01-B17 biases and *cognitive consistency, cognitive inconsistency, cognitive dissonance of the state actor* [2]). Then with the help of the relation (2) we establish the relevant rates for the hierarchical system of objectives of the state decision maker.

$$\max(r_{ik}, w_i) = \begin{cases} r_{ik}, r_{ik} \geq w_i \\ 0 \end{cases} \tag{2}$$

For the calculation of the final choice option of each action we define by the relation (3) the option as below in which only the rates that exceed the weight of the objectives are summed and prove that the respective action is viable to reach the proposed objectives.

$$FC_k(r_{1k}, r_{2k}, ..., r_{mk}, w_1, w_2, ..., w_m) = \sum_{i=1}^{m} \max(r_{ik}, w_i) \tag{3}$$

The methods of calculating the rates will be discussed in the results section. As limitations, this research does not take into account the evaluation of the external conditions for limiting the expansion of the state actor defined by Gilpin: increasing costs of political dominance, loss of economic and technological leadership [1].

3. RESULTS

In the specialized literature, the purely rationalist approach is widespread. By this approach we consider a step forward in the process of knowledge of cyberwarfare which proposes an alternative approach that wants to bring the process of analysis closer to the reality of the research object. In order to deepen research in calculating the rates of the specific poliheuristic decision matrix, **the first element is to establish the management model in foreign policy**, we call on the qualitative and quantitative hybrid study methodology of the international events proposed by Sokolowski and Banks [5] which takes place in three stages: 1. A historical research of the study event; 2. Investigation of the event through statistical indicators; 3. Defining a quantitative predictive system for modeling statistical indicators and predicting the evolution of the event. Validate approaches through qualitative and quantitative comparisons that describe the event being investigated. As we stated in the introduction to this article, the phenomenon of cyberwarfare is not a singular one but we consider it an intrinsic one of the expansion of military, economic, cultural, etc. nature of a state actor. That is why in this research we approach the flexible or massive expansion model exhibited by authors such as Robert Gilpin [1], Jack Snyder [11], Paul Kennedy [12]. That is why we use the "Gilpin Test" in the research of the decision of flexible or massive expansion / over-expansion of a state actor. Gilpin explains this phenomenon through five internal conditions defined according to: cost for national security c_{CNS}; cost for private and nonmilitary public c_{CPNP}; cost for productive investment c_{CPI} and the level of corruption or good internal governance. The five conditions are the following [13]:

GT1. The gross domestic product suffers from a great decrease

$$\frac{\Delta c_{GDP}}{\Delta t} \downarrow < 0 \tag{4}$$

GT2. Military expenditures rise too sharply compared to the other two costs

$$\frac{\Delta c_{CNS}}{\Delta t} >> \frac{\Delta c_{GDP}}{\Delta t} \; AND \; \frac{\Delta c_{CNS}}{\Delta t} >> \frac{\Delta c_{CPNP}}{\Delta t} \; AND \; \frac{\Delta c_{CNS}}{\Delta t} >> \frac{\Delta c_{CPI}}{\Delta t} \tag{5}$$

GT3. Gross domestic product has a lower growth than non-military public and private expenditures

$$\frac{\Delta c_{CPNP}}{\Delta t} >> \frac{\Delta c_{GDP}}{\Delta t}; \tag{6}$$

GT4. Innovative investments being lower than the other two military and non-military costs, the structural change of the type of economy cannot be achieved because

$$c_{CPI} < c_{CPNP}\ AND\ c_{CPI} < c_{CNS} \tag{7}$$

GT5. High degree of corruption depending on the global indicator of good governance or the perception of corruption.

For the study of the compound phenomenon of expansions of a nation-state composed of cyberwarfare, we propose a polyurethane matrix below in which the two types of flexible expansions (which do not satisfy the Gilpin test) and the massive ones (which satisfy the Gilpin test) are paired with the six types of cyberwarfare strategies proposed by the Cyber Operations Tracker of Council on Foreign Relations. The critical dimensions remain the same as in Table 2. (FE = Flexible Expansion, ME = Massive Expansion).

Table 3. Cyberwarfare poliheuristic approach of nation-state expansion based on "Gilpin Test"

	Flexible Expansion			Massive Expansion			Weights
	DDOS	...	Doxing	DDOS	...	Doxing	
Civil society	r_{11}^{FE}	...	r_{16}^{FE}	r_{11}^{ME}	...	r_{16}^{ME}	w_1
Government	r_{21}^{FE}	...	r_{26}^{FE}	r_{21}^{ME}	...	r_{26}^{ME}	w_2
Military	r_{31}^{FE}	...	r_{36}^{FE}	r_{31}^{ME}	...	r_{36}^{ME}	w_3
Private sector	r_{41}^{FE}	...	r_{46}^{FE}	r_{41}^{ME}	...	r_{46}^{ME}	w_4
Final choice	FC_1^{FE}	...	FC_6^{FE}	FC_1^{ME}	...	FC_6^{ME}	

The poliheuristic decision matrix is shown in Tabel 3 above. After identifying the B01-B17 biases, the dimensions of the decision matrix, the actions we carry out a historical analysis of the expansion event combined with cyberwarfare and we identify as statistical indicators mentioned above: investments in economic innovation for the private sector dimension, military expenses for the military dimension, non-military public and private expenditures for the civil society dimension, the indicator of good governance for the government dimension, and the GDP for establishing the type of flexible or massive expansion if the conditions of GT1-G5 are verified. In the second stage of the decision to establish the decision rules, before applying the relations (1), (2) and (3) we calculate using a qualitative-quantitative type predictive approach of the post-event statistical indicators, knowing them. those pre-event through time series, neural networks, or other learning machines. The indicators used are: indicator of good governance from the World Bank [14], military expenses / non-military public and private expenses / investments in economic innovation / GDP from the CROSS-NATIONAL TIME-SERIES DATA ARCHIVE data bank [15]; cyber indicator of the number of attacks of the Cyber Operations Tracker of Council on Foreign Relations […].

$$r_{ij}^{cyber}(\Delta N) = k, k \cdot \frac{N_{Max} - N_{Min}}{10} \le \Delta N \le (k+1) \cdot \frac{N_{Max} - N_{Min}}{10}, k = \overline{1,9} \tag{8}$$

$$r_{ij}^{non-cyber}(\Delta c) = K, K \cdot \frac{c_{Max} - c_{Min}}{10} \le \Delta c \le (K+1) \cdot \frac{c_{Max} - c_{Min}}{10}, K = \overline{1,9} \tag{9}$$

$$r_{ij}^{X} = \begin{cases} r_{ij}^{cyber} \\ r_{ij}^{cyber} \cdot \alpha_{ij} + r_{ij}^{non-cyber} \cdot (1 - \alpha_{ij}), \alpha_{ij} \in (0,1); X = \overline{FE, ME}, \\ r_{ij}^{non-cyber} \end{cases} \tag{10}$$

Where r_{ij}^{X} is the final rate in the decision matrix used in relations (1) - (3); r_{ij}^{cyber} is a rate assessed based on quantitative predictions of cyber attacks from the Cyber Operations Tracker of Council on Foreign Relations database; and $r_{ij}^{non-cyber}$ is the rate resulting from the quantitative prediction of any type of economic cost or indicator of good governance; parameter α_{ij} further noted α for simplicity is a subunit and positive percentage parameter and is considered to be known in advance and can be analyzed based on previous case studies using the hybrid methodology proposed by Sokolowski and Banks by analyzing historical events.

Thus he decides whether for a particular type of cyber attack in a type of mass or flexible expansion, cyber operations or economic mechanisms or good governance were more important. The rates of the decision-making poliheuristic matrix are defined by the relations (8) - (10) above, and the rate variant is decided from the analysis of past facts that are the basis for the prediction of future actions. If the decision was purely economic or cyber, one opts for the rate that results only from the economic cost specific to the size of Civil society, Military, Private sector. For the Government dimension we use the indicator of good governance and for the cyber rate the number of attacks on a certain dimension. If one takes the analysis of the facts both the cost / governance and cyber parameters, the parameter α is used which is also used in the prediction, considering it a constant. *Thus, after establishing the rates, the second step is determined* **determining the decision rules** *and applying the relations (1) - (3) to establish the hierarchy of the final decisions of the actions of the state actor.*

This research brings as novelty the involvement of international statistical parameters in the calculation of decision rates as a possibility to evaluate the cognitive consistency [...] of the decision maker in foreign policy actions using the hybrid evaluation method of international events proposed by Sokolowski and Banks. This is a novelty besides the models of decision rules proposed by Mintz of conjunctive, disjunctive or elimination type by aspect [2].

The results presented based on the use of World Bank statistical indicators, CROSS-NATIONAL TIME-SERIES DATA ARCHIVE, Cyber Operations Tracker of Council on Foreign Relations and relations (1) - (3) and (8) - (10) present a flexible approach which is extendable to other areas of foreign policy research and security studies and management operations in general. Compared to other approaches to the analysis of foreign policy, which involve establishing decision rates only based on the analysis of historical facts [5], which can mix the analysis of the biases of the political actors with the researcher's biases, the approach proposed by us by calling on statistical indicators and operationalizing their objectives by indicators, these disruptive factors are largely removed. By the approach proposed in this research in the analysis of a certain action, one can establish the ratios that were the basis of the decision of an actor by establishing a percentage for the variable α which explains the importance that the decision maker gave to a certain objective in the hierarchy of objectives and the importance of prejudice from the list B01-B17 that were the basis of that option. The poliheuristic analysis used to analyze the foreign policy decision that accompanies the expansion policy of a nation state using cyber means goes beyond the purely rationalist approach which dominates the specialized analysis for the most part today, presenting a revolutionary approach through the appeal of the decision in two stages, to the list of biases B01-B17 proposed by Mintz and the decision-making matrix as alternatives to options based purely on game theory. According to the hybrid approach of the events proposed by Sokolowski and Banks and applicable in the evaluation of the rates, we obtain a high percentage of their evaluation in the most objective construction of the decision of a state political actor based on international statistical indicators. As we exemplify above, the approach presented in this research has the greatest flexibility and applicability in areas such as international relations and foreign policy, economic policies, security studies, management operations in general, etc.

CONCLUSIONS

In the researches that dominate the study of international relations of the dominant cyberwarfare are the rationalist theses, which try to highlight only focus on the end-result but neglect the internal constraints of the state actor but also his prejudices that can affect a rational decision and as a result can affects a good quality analysis.

That is why the logical step was to call for an approach such as foreign policy analysis, in our case the poliheuristic paradigm [2], to overcome these impediments. Thus, we appealed to the poliheuristic paradigm for the deep study that it offers to the decision maker's prejudices and the two-stage approach that allows the introduction of quantitative type calculations in the second stage of the study. Thus, our research has taken a step forward by integrating the qualitative-quantitative hybrid approach proposed by Sokolowski and Banks in evaluating the rates underlying the hierarchy of final decision rules.

Our research has presented how biases can influence the decision by reordering according to the hierarchy of the objectives that it has and the model to influence the order of the final decision rules specific to the second stage of decision. Thus, through the proposed approach, the researcher can get closer to the cognitive mechanisms underlying the decision and the cognitive inconsistency or inconsistency that Mintz speaks of [2]. As important contributions we can name the implication of a qualitative-quantitative hybrid approach for evaluating the rates of a decision matrix by using common factual-historical analysis in establishing the hierarchy of biases and quantitative predictive models such as time series and complementary analysis mechanisms. The novelty of this article in the field of cyberwarfare security studies is represented by the internal constraint that the state system can bring to achieving the foreign policy objectives and limiting the decision-makers' vision due to their own biases. The applications can be counted besides the classic ones in the studies of security and the foreign policy can be extended to the researches in variants domains of the management or of the different economic and social policies. About the limits of the research the most important is probably the interference between the historical approach from the beginning of the qualitative-quantitative hybrid analysis and the operationalization of the concepts by the researcher. Something that has to be treated very carefully and checked several times. As a future activity we intend to improve the limitation mechanism mentioned above and the possibility of an objective approach of the α parameters in the decision rules.

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HYBRID COMPOSITE MATERIALS FOR BALLISTIC PROTECTION. A NUMERICAL ANALYSIS

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Abstract: *Starting from the basic element of projectile-armor competition, approaching impact problems, applying the optimal elements synthesis in evaluation and results integration, the paper intends to provide a comparative analysis between an armor plate made of conventional materials and a hybrid armor plate.*

In this paper, the behavior of a steel armor plate and a multilayer plate are numerically analyzed in terms of dissipated energy. The structure's numerical model is validated, first and foremost by experimental tests performed in the range, observing the projectile yielding mode, and the impact on the armor plate simultaneously. The dissipated energy on the multilayer plate is further determined for certain armor inclination angles, and then compared with the steel plate.

Keywords: *armor plate, projectile, hybrid armor plate, multilayer plate, impact, amphibious armored vehicle, finite element modelling, sandwich structure, numerical analysis, specific weight, kinetic energy, dissipated energy.*

1. INTRODUCTION

The armed conflicts and social movements that characterized the 20th century beginning have conditioned the occurrence and diversification of both individual protection (bulletproof vest, ballistic helmet) and the automotive industry in equipping armed forces with combat machine, properly equipped to ensure the protection of both the crew who operate them and the fighters.

Such a type of technique is not only characterized by the degree of mobility and its ability to move smoothly on any type of terrain, but also by the protection against the firearms.

In order to achieve the protection against firearms, the aim is to manufacture a relatively resistant armor, but also with lower weight. At the first sight, these two aspects of the technology – mobility and armored protection – sound to be contradictory, due to the fact that in order to achieve a higher impact strength, the armor plate must have a maximum layer thickness, and furthermore to obtain a high mobility, the armor plate must be as light as possible. This obstacle can be defeated by using materials that provide greater specific stiffness and strength-to-weight ratio. Therefore, the materials selection is an important criterion for weight optimization, such as the use of improved composite materials.

The impact strength performance of such light structures made of composite materials is an important design criterion in the armor protection area.

2. DIRECTION TO IMPROVE ARMOR PROTECTION, TRANSPOSED IN PRACTICE, BOTH NATIONALLY AND INTERNATIONALLY

From the armed conflicts that have evolved over time perspective, it can be concluded that the main effort in the armor protection area, have been directed in manufacturing of new modern combat machines types, as well as towards the modernization of some technologies used for manufacturing their main components and subassemblies.

Contemporary armored vehicles are equipped with conventional, homogenous shielding materials, which are generally made of steel or aluminum alloys. (Fig. 1)

Today tendency, both nationally and internationally is to research new armor plates that are lighter as possible and resistant to multiple threats.

The actual armored vehicles efficiency requirements, including rapid displacement, increased drive distance and improved ballistic protection, significantly contribute to the increasing armored vehicles efficiency, and as a result, higher survival rate on the battlefield.

The present direction is to insist on obtaining armor plates made of lighter multilayer composite materials, which protect against the armor piercers and cumulative ammunition effects, such as reducing overpressure and explosion amplitude.

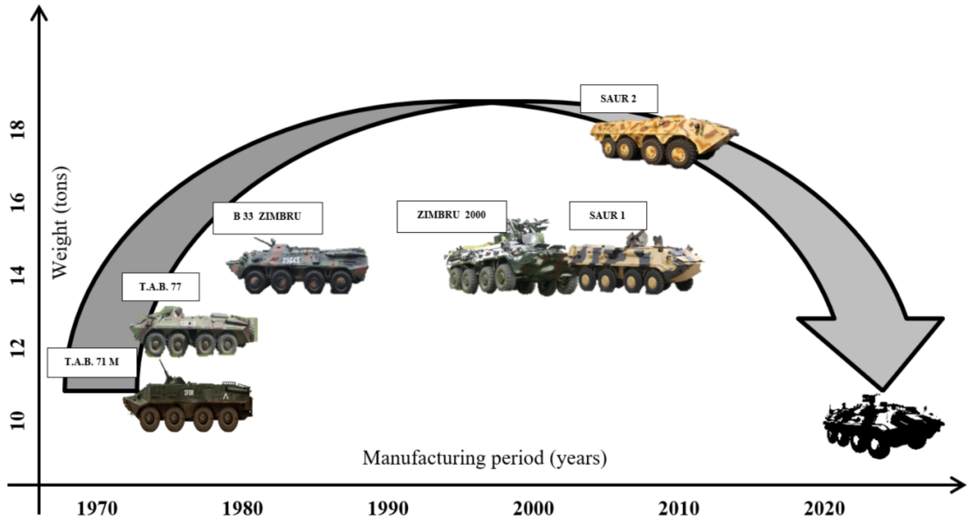


FIG. 1. The direction of reducing the weight of armored vehicles

3. THE MAIN CHARACTERISTICS AND DESTINATION OF AMPHIBIOUS ARMORED VEHICLES

The amphibious armored vehicles are means of transportation and combat for the infantry group, that ensure superior mobility, high fire power and adequate protection against light infantry weapons projectiles (5,45x39 mm or 7,62x39 mm caliber) and shrapnel, and can be exploited day and night, regardless of time and season or terrain.

In the table below are presented some technical and tactical characteristics, namely the armor plates thickness and their arrangement inside of the frame partitions, for the amphibious armored vehicles that equipped the Romanian army.

Global military operation has seen the demand increase for a vehicle that bridges the gap between deployable light forces, with their inherent low survivability, and more survivable heavy forces that are difficult to deploy and have high life cycle costs. PIRANHA IV has been developed in response to this need and takes advantage of the latest technologies to deliver class leading survivability, capacity and mobility with low through life costs and inherent growth capacity [3]. The PIRANHA V is a wheeled armored combat vehicle, being considered one of the most advanced platform for troop transport equipped with modern ballistic protection systems and fire power, at the highest endowment level available in NATO. It has an all-welded steel armor hull with integrated add-on composite modular armor system. This system provides unparalleled all round protection, particularly in the wheel well area that has traditionally been vulnerable to IEDs on armored combat vehicles.

Table 1. The thickness of the armor plates for the T.A.B. that equipped the Romanian army [1, 2]

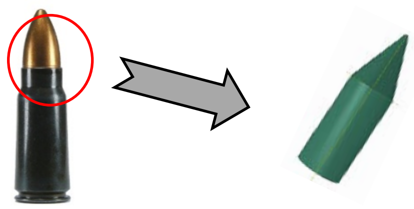
Parameter	TAB-71	TAB-71M	TAB-77	TAB-77-30M
Thickness of the armor plate (mm)	***Arrangement***			
4	ceiling and ceiling shutters		the ceiling of the energy room, the wings of the niches	
5	-	-	ceiling and ceiling shutters	
6	upper and lower sides, upper front, back welded			
8	-	-	the sides of the niches	
10	windshield		windshield lower front	
12	turret shield		-	-
13	lower front		plate angle on the lateral sides	
20	vertical face plate, observation deck			

The armor has a very high influence on the armored vehicle weight. As first method of increasing the ballistic protection, is represented by the armor plate thickness increase, primarily this solution was preferred. Weight gain leads first to high vulnerability considering the mobility point of view, proportionally increasing the engine power. The requirements to increase the armor thickness lead to the occurrence of the *equivalent armor* concept, referred to the armor plate angle, that provides better protection (equivalent, reported to the same real thickness) and, through the constructive solution adopted (maintaining the equivalent thickness, therefore of the equivalent ballistic protection, with the plate leaning), you can get an vehicle mobility increase [4].

4. THE CONFIGURATION AND THE CONSTITUENT MATERIALS USED TO APPROACH THE IMPACT PROBLEM

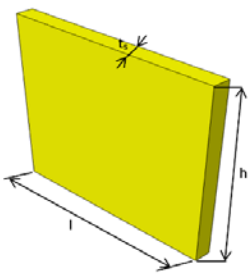
The elements used in this paper, in approaching the impact problem are the penetrating object (projectile) and the target (armor plate).

The projectile belongs to the 7,62 mm light infantry weapon (7,62x39 mm cartridge, steel core bullet, steel tube) [5], and the armor plate is made of steel used in armored vehicles manufacturing, for protection against high-impact projectiles [6].



(a) (b)

FIG. 2. The projectile used in analysis a) 7,62x39 mm cartridge [5], b) 3D model 7,62x39 mm cartridge



l = 100 mm- the armor plate lenght
h = 100 mm - the armor plate height
t_s = 8 mm – the armor plate thickness

FIG. 3. The geometrical parameters of the armor plate model

5. NUMERICAL ANALYSIS

The numerical analysis is based on a FE model, developed in commercial software Abaqus, the structure being analyzed from a dynamic point of view. In order to perform the analysis, it is necessary to obtain the material constitutive low. This law allows numerical simulation of the technological processes that involve high rates of strain, as well as impact problems. The material model statement establishes the relationship between stress, specific strain, rate of strain and the temperature and involve the material parameters knowledge.

5.1 Geometry and mesh type. The geometrical configuration numerically analyzed are shown in the figure below, the modification being imposed by tilting the armor plate under a certain angle of incidence, therefore providing a better protection, through the adopted constructive solution and obtaining an increased vehicle mobility.

(a) (b)

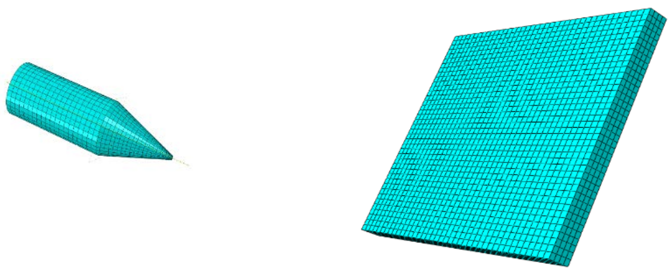
FIG. 4. Armor plate tilt angle a) initial position - angle of incidence 0°, b) modified position- angle of incidence 20°

Table 2. Angle of incidence α

Angle of incidence α [°]	0	20	40	50	60

The geometry of the two elements used in the analysis (projectile and the armor plate) was created selecting type *Solid*, to simulate as much as possible the properties of the physical model properties.

The mesh sensitivity in the projectile was studied by considering the element size of 0,75 mm, for the target an element size of 2,5 mm, and for the width a 4 elements size.



(a) (b)

FIG. 5. Finite element model a) projectile, b) armor plate

5.2 Material properties. For the analyzed model, a projectile impact speed of 690 m/s was chosen, taking into account that the shooting on the target distance is 25 m. [7]

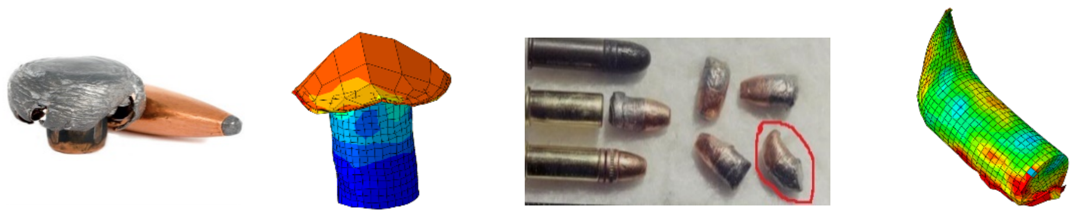
The Johnson-Cook constitutive model material parameters were used to predict the ballistic steel target performance and model failure in order to predict the material damage behavior.

Table 3. Material parameters for the projectile and armor plate [8, 9]

Parameter	**Unit**	**Notation**	**Value (projectile)**	**Value (armor plate)**
Modulus of elasticity	N/m^2	E	202×10^9	210×10^9
Poisson´s ratio	-	υ	0,32	0,33
Density	Kg/m^3	ρ	7850	7850
Johnson-Cook plasticity constitutive model	N/m^2	A	2700×10^6	980×10^6
	N/m^2	B	211×10^6	2000×10^6
	-	n	0,065	0,83
	-	c	0,005	0,0026
	-	m	1,17	1,4
	-	$\dot{\varepsilon}_0$	0,0001	0,0001
	K	T_m	1800	1800
	K	T_{tr}	293	300
Johnson-Cook damage constitutive model	-	D_1	0,4	0,05
	-	D_2	0	0,8
	-	D_3	0	-0,44
	-	D_4	0	-0,046
	-	D_5	0	-2,9

6. RESULTS AND DISCUSSION

The FE model is validated by comparing deformed the projectile shape after impact by experimental determination, in the real firing tests carried out in the firing ranges, with the results obtained numerically in the present work. At the same time, the evaluation of the hole formed in the target is also considered.



(a) (b) (c) (d)

FIG. 6. The deformed shape of the projectile after impact a) [10] and c) experimental results, b) [11] and d) numerical results

In “FIG. 6.” a) and b) is presented the projectile deformation mode at a frontal impact (angle of incidence - 0°). It can be observed that the deformation mode obtained by the experimental determination and the numerical solution is quite similar, both models are very much deformed, being in a crushing state.

In “FIG. 6.” c) and d) is presented the projectile deformation mode at an angle of incidence dissimilar to 0°. It can be observed that the deformation mode obtained by the experimental determination and the numerical solution, same as in the previous case, is quite similar, both models having a specific strain when the projectile bounces on impact.

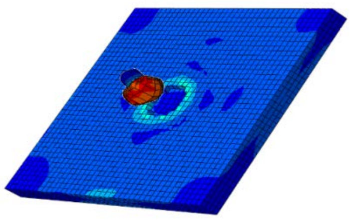


FIG. 7. Deformation of armor plate

FIG. 7. illustrates the armor plate behavior in a frontal impact. It can be observed that the projectile pierces into the armor plate without penetrate it.

According to the technical and tactical specification of amphibious armored vehicles, their armor provide protection against light infantry weapons, resulting that the numerical solution obtained behaves like the physical model. Therefore, it is concluded that FE model predicts very well the experimental behavior, both for the projectile and the armor plate.

7. IMPACT STUDY FOR A HYBRID MULTILAYER PLATE

Based on the validated FE model, an impact analysis was performed on a hybrid multilayer plate, intending to obtain a lighter armor, but at the same time with the properties to respond in the front of the current threats.

The same projectile, impact velocity and angles of incidence will be also applied in this case. During the simulation, the multilayer plate behavior will be compared with the obtained steel plate results.

7.1. The configuration and the constituent materials. Further, a multilayer armor plate is proposed, in which architecture several layers of materials are grouped: *first layer* is made of ceramic composite materials (alumina 99,5%); the *second layer* is made of fiber based composite materials (glass fiber reinforced polymer – gfrp); the *third layer* is a sandwich structure (core and shell acrylonitrile butadiene styrene – abs); the *fourth layer* is made of metal (al7075-t651).

Adhesive was used to join the plates. The main purpose of the adhesive is to maintain homogenous the armor before and after impact as well as to absorb the deformation and delamination.

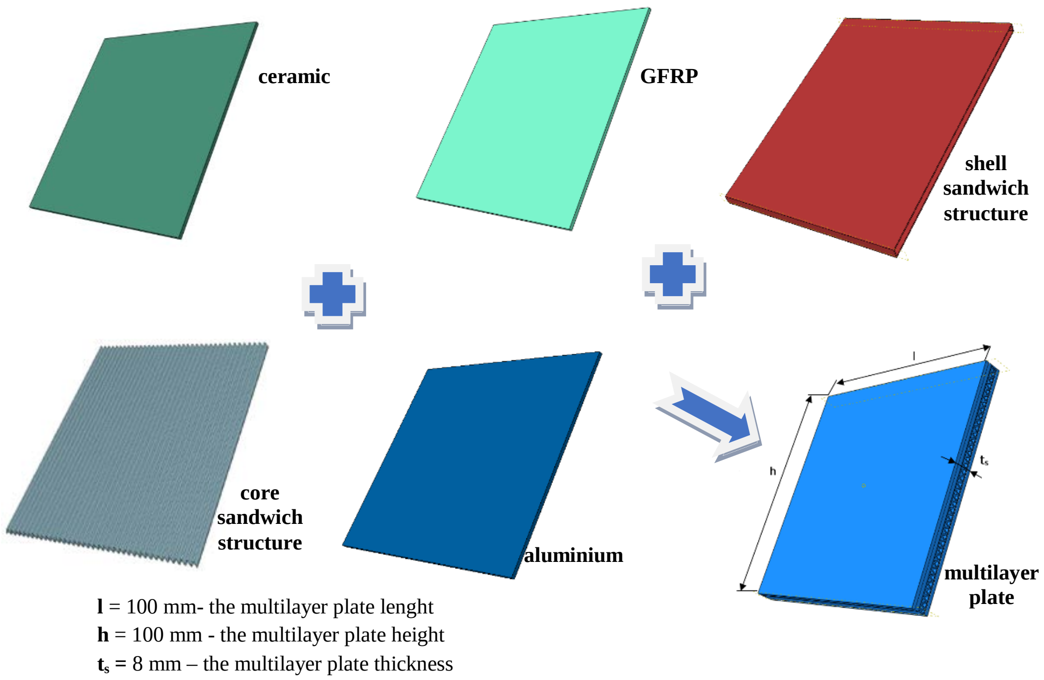


FIG. 8. The geometrical multilayer plate model parameters

7.2 Geometry and mesh type. The geometry of the elements used in the analysis was created of *solid* type, and in addition the ceramic plate and the gfrp plate were modeled as composites.

The mesh sensitivity in the ceramic plate, GFRP plate and aluminum plate was studied by considering the element size of 2,5 mm, and width of 2 elements. For the shell sandwich structure an element size of 3 mm, and a width of 3 elements were considered in. In the case of core sandwich structure, an element size of 3 mm, and a width of 2 elements were considered.

FIG. 9. Finite element model for multilayer plate

7.3 Material properties. The projectile was impacted normally with an incidence velocity 690 m/s, counting that the shooting distance on the target is 25 m.

In order to simulate the behavior of the material on impact, the Johnson-Cook plasticity and damage constitutive model for the material parameters was used on sandwich structure and aluminum. For the composite materials, the Hashin criterion was applied.

Table 5. Material parameters for Al and ABS [12, 13, 14]

Parameter	Unit	Notation	Value (Al)	Value (ABS)
Modulus of elasticity	N/m^2	E	$71{,}7 \times 10^9$	$2{,}9 \times 10^9$
Poisson´s ratio	-	υ	0,33	0,422
Density	Kg/m^3	ρ	2810	2810
Johnson-Cook plasticity constitutive model	N/m^2	A	520×10^6	39×10^6
	N/m^2	B	477×10^6	48×10^6
	-	n	0,52	1,5
	-	c	0,0025	0,544
	-	m	1,61	0,879
	-	$\dot{\varepsilon}_0$	0.0005	0,00081
	K	T_m	893	513
	K	T_{tr}	293	300
Johnson-Cook damage constitutive model	-	D_1	0,096	0
	-	D_2	0,049	0
	-	D_3	3,465	0
	-	D_4	0,016	0
	-	D_5	1,099	0

Table 6. Material parameters for Alumina and GFRP [15, 16, 17]

Parameter	Unit	Notation	Value (Alumina)	Value (GFRP)
Young modulus	N/m^2	E_1	$20{,}44 \times 10^9$	13×10^9
	N/m^2	E_2	$8{,}9 \times 10^9$	13×10^9
	N/m^2	E_3	$8{,}9 \times 10^9$	$2{,}4 \times 10^9$
Shear modulus	N/m^2	G_{12}	$1{,}64 \times 10^9$	$17{,}2 \times 10^9$
	N/m^2	G_{13}	$1{,}64 \times 10^9$	$17{,}2 \times 10^9$
	N/m^2	G_{23}	$3{,}03 \times 10^9$	$17{,}2 \times 10^9$
Poisson coefficient	-	υ_{12}	0,31	0,1
	-	υ_{13}	0,31	0,3
	-	υ_{23}	0,49	0,3
Density	Kg/m^3	ρ	1230	1800
Longitudinal tensile strength	N/m^2	Hashin criterion	$1{,}145 \times 10^9$	$0{,}32 \times 10^9$
	N/m^2		$0{,}13 \times 10^9$	$0{,}32 \times 10^9$
Longitudinal compressive strength	N/m^2		$0{,}65 \times 10^9$	$0{,}24 \times 10^9$
	N/m^2		$0{,}65 \times 10^9$	$0{,}24 \times 10^9$
Shear strenght	N/m^2		$0{,}34 \times 10^9$	$0{,}14 \times 10^9$
	N/m^2		$0{,}34 \times 10^9$	$0{,}14 \times 10^9$

8. RESULTS. DISSIPATED ENERGY

In this paper study, dissipated energy (lost) is represented as the difference between impact kinetic energy and kinetic energy remaining after impact.

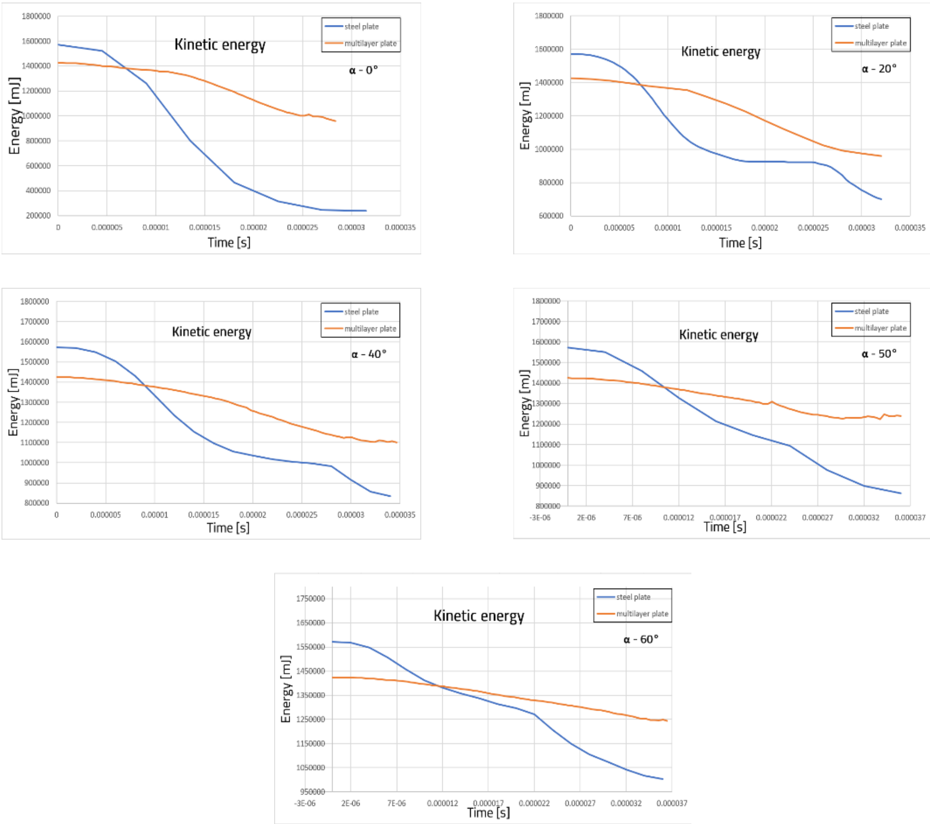


FIG. 10. Kinetic energy for the steel plate, respectively the multilayer plate

The energetic values for kinetic energy are calculated for t=0,000035 s. In the graphs above can be observed that the dissipated energy in the steel plate case is higher than in the multilayer plate case. This is happening due to higher weight of the steel plate compared to the multilayer plate.

To analyze the armor plates behavior at impact in terms of dissipated energy the E_d/m ratio will be used, where *m* represents the armor plates weight.

To calculate dissipated energy will be used the relation:

$$E_d = \frac{E_{kmax}}{E_{kmin}} \quad (1)$$

where

E_{cmax} represents the kinetic energy at the moment of impact (t = 0 s);

E_{cmin} represents the kinetic energy at the moment t = 0,000035 s.

The following values resulted from the calculation:

$$E_{Sd}(\alpha = 0°) = 2123.3\frac{J}{kg}$$

$$E_{Md}(\alpha = 0°) = 4006.9\frac{J}{kg}$$

$$E_{Sd}(\alpha = 20°) = 1389.6\frac{J}{kg}$$

$$E_{Md}(\alpha = 20°) = 4014.9\frac{J}{kg}$$
$$E_{Sd}(\alpha = 40°) = 1176.4\frac{J}{kg}$$
$$E_{Md}(\alpha = 40°) = 2784.1\frac{J}{kg}$$
$$E_{Sd}(\alpha = 50°) = 1130.1\frac{J}{kg}$$
$$E_{Md}(\alpha = 50°) = 1590.9\frac{J}{kg}$$
$$E_{Sd}(\alpha = 60°) = 907.3\frac{J}{kg}$$
$$E_{Md}(\alpha = 60°) = 1549.4\frac{J}{kg}$$

For solving numerical calculation was considered a weight of 0,628 kg. was considered for the steel plate and 0,116 kg. for the multilayer plate.

Based on results it can be observed a higher value of E_d/m ratio in all five analyzed cases, in the favor of the multilayer plate.

Referring to the structure weight can be concluded that the multilayer plate possesses a better behavior at impact, compared to the steel plate, in terms of dissipated energy.

CONCLUSIONS

Based on validated numerical model, a parametric study was performed to analyze the dissipated energy. The multilayer armor plate investigated in the paper is proved to be a competitive structure in terms of dissipated energy. This behavior is an added value in addition to its considerably light weight compared to a steel armor plate. The plate consists of four layers kept homogenous by an adhesive used in ballistic protection structures.

Although the structure has limited advanced over conventional armor plates, it can be used where light weight and dissipated energy are important.

Mixed armors made of composite, glass fiber, polymer, sandwich structure and more, seem to form a very efficient shield against low and high velocity impact, since they combine low density, high hardness, high rigidity, strength in compression, lightweight and ductility.

However, apart from material type, shape criterion is also important and it may represent an added advantage. An example of near optimal use of material is given by the sandwich concept, where the bending stiffness of the structure is increased by placing a lightweight and thicker core between two thin and stiff face sheets while the weight is negligibly increased. The continuing research on improving the overall mechanical performance of sandwich structures focuses also on developing novel core configuration, made of composite materials, in order to gain an improved mechanical behavior of the core. Although many of these structures provide competitive weight specific strength and stiffness, their main drawback is related to manufacturing steps which are often complicated and difficult to be integrated within a continuous production line. The more recent development of additive manufacturing technologies allows generating complicated and efficient cellular shapes but on a limited scale yet.

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MODELING IN OPERATIONAL RESEARCH

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Abstract: *In order to be able to develop the organization at a high level of efficiency, every structure that makes up the organization should grow relatively at the same pace as the other structures or compartments.*

Hence the need to model the organizational system at the virtual level and to simulate the different stages of the organization system development taking into account the disturbing factors caused by the internal and the external environment of the organization.

Keywords: *operational research, mathematical modeling, model validation, implementation*

1. INTRODUCTION

As we know, operational research is based on the mathematical analysis of the various internal and external environmental issues of the organizational society as a whole, modeling and validating various organizational issues by finding an optimal solution.

The way they are defined, operational research has constituted the basis for the development of the organizational culture as a whole, since there have been identified many problems that have arisen from the multi-level segregation of work in an organizational unit.

In order to be able to develop the organization at a high level of efficiency, every structure that makes up the organization should grow relatively at the same pace as the other structures or compartments. However, experience says that what is good for a compartment may be to the detriment of other compartments or structures of the organization. Thus, the allocation of resources, which is more and more difficult to achieve in case of a complex organization, is streamlined, it leading ultimately to an increase in the efficiency of the organization by intersecting the goals of all compartments or structures.

Hence the need to model the organizational system at the virtual level and to simulate the different stages of the organization system development taking into account the disturbing factors caused by the internal and the external environment of the organization.

2. HISTORICAL REFERENCE

Operational research has achieved an impressive progress during the Second World War at the same pace with the increasing complexity of military operations, from the tactical level to the operational and strategic level.

Thus, in support of the Armed Forces, top researchers from all areas were invited to model and simulate various complex stages to make military action more effective, taking into account several external factors of political, economic and social nature.

All these challenges, which the operational research attempts to solve are due to the influence of environment on the organization and are aimed at finding solutions to the increasingly complex challenges of the organizational environment challenges that originate in the unprecedented development of industry and applied sciences.

After World War II, operational research has developed primarily due to the migration of the majority of specialists in the field of military operational research in the field and respectively in the civilian economic and social environment. The industrial boom at the end of the Second World War, based on various economic and social recovery plans for the defeated nations, has generated similar operational research problems in the civilian organizational environment, and military specialists in operational research have been the ones to settle the foundations of operational research in the civilian field.

The organizational culture benefited from the help of military specialists, but a significant contribution to it was facilitated by the unprecedented technological revolution in the IT area. The increased computing power of electronic computers has shortened the arithmetical computational time in the simulation process, which has led to increased work productivity in the field of operational research.

Nevertheless, the real change occurred in the early 1990s and culminated in the early 21st century when personal computer development accelerated, and the development of software packages dedicated to Microsoft Excel operational research began to provide solutions to almost all new challenges in the operational research environment.

The last cornerstone was the development of portability by using portable devices such as laptops, notebooks or personal assistants used on hardware architectures that use multicore processors with artificial intelligence.

3. WHY OPERATIONAL RESEARCH?

Definition: *Operational research is the study of mathematical models of complex engineering and management problems and perspectives of implementation and use of possible optimal solutions in this study* [1].

Because, as the title says, when there are concerns or research on improving the organizational culture, they need to be operationalized, or else the problem formulated by careful observation must be solved by constructing a mathematical model that will reveal the essence of the problem or the hypothesis from which it started.

Therefore, not only the problem has to be solved, but also the sum of unknown factors that derive from the phases of the very modeling in the operational researches; subsequently we can enumerate 4 phases of modeling (FIG. 1):

1. Knowledge of reality in the studied organization, in order to improve the decisional information mechanism, to define the issue of interest and to collect important data.

2. Formulation of a mathematical model of problem representation. This operation, in most of practical cases, consists of applying a classical modeling tool chosen from the extremely varied range available to us due to the theory of operational research. In such situations, the analyst’s ability is to establish the correspondence between the reality and the modeling tool known in the specialized literature.

3. Development of a procedure or technique based on computer simulation conclusions in order to find solutions to the mathematically modeled problem.

4. Testing and implementing the model, refining the results and making decisions.

An axiom-based model (axiomatic system) comprises:

- the axioms of the system, representing sentences expressed in mathematical form, usually very few, which contain some very general truths on the phenomenon that is being modeled, so general that all concrete and particular findings can be deduced from the general ones;

- rules of inference, representing rigorous prescriptions, the only ones admitted in the system, from which one passes from axioms to theorems, or from already demonstrated theorems to new ones;
- theorems, i.e., more or less particular sentences, mathematically expressed, deduced by step-by-step inference rules from axioms and expressing the properties of the modeled phenomenon.

When in the axiomatic modeling process the concepts to be used are defined in a limiting manner, i.e. a list of the mathematical notions and operations admitted in the system is given from the beginning; a superior form of axiomatic system called formal system is obtained.

Formal systems are still very little used in science and even less in the disciplines of organization and economic leadership.

Axiomatization and, ultimately, formalization, represents the future in mathematical modeling, due to the exceptional rigidity they introduce, the considerable diminution of the elements of intuition and arbitrariness, which, although much less than in the non-mathematical models, are still present in the axiomatized mathematical modeling .

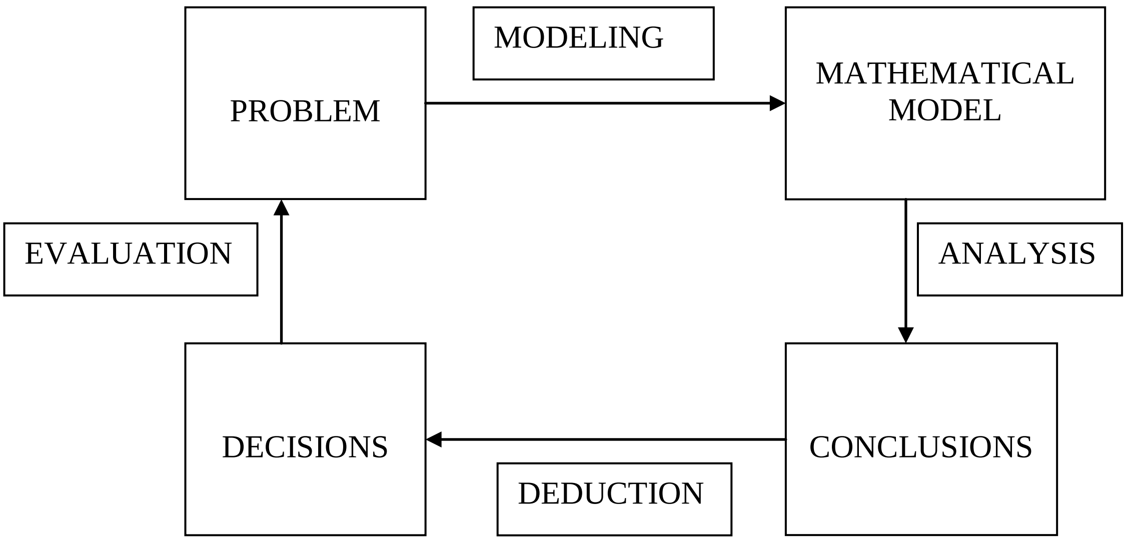

FIG. 1 Operational research method

4. CHARACTERISTICS OF MODERN MODELING INSTRUMENTS

Given the fact that we know the decision problem, the process will begin with formulation or modeling. We define the variables and quantify the relationships necessary to describe the relevant behavior of the organizational system. Then the analysis comes. We apply our mathematical knowledge and skills to see what the mathematical model suggests. Note that these conclusions are extracted from the model, not from the problem it is intended to represent. In order to complete the process, we must participate in inference, that is to say that the conclusions drawn from the model are sufficiently significant to deduce the decisions for the person or persons who wish to solve this problem.

Often an evaluation of the decisions deducted in such a manner proves to be inadequate or extreme and cannot be used for implementation. Permanent thinking leads to revised modeling, in a continuous loop.

Thus, the definition of the problem becomes the most important phase, defining the new operational requirements of the organization on the basis of the hypothesis.

This process of defining the problem is crucial, because it is very important how it affects us and how relevant the conclusions of the study are.

After defining the problem of the decision-maker, the next step is to reformulate this problem in a form that is convenient for analysis. The conventional approach to operational research is to build a mathematical model that represents the essence of the problem. Before discussing how to formulate such a model, we first analyze the nature of models, in general, and the mathematical models, in particular.

Models or idealized representations are an integral part of everyday life. Common examples include aircraft models, portraits, globes, and so on. Similarly, models play an important role in science and business, illustrated by atom models, genetic structure models, mathematical equations describing physical motion laws or chemical reactions, graphs, organizational diagrams, and industrial accounting systems.

Such models are invaluable for abstracting the subject matter of the investigation, presenting interrelationships and facilitating the analysis.

The next step is to check if the mathematical model is accurate enough, reflecting the initial hypothesis in order to define the real problem.

Next, validation of the mathematical model must be achieved by experiments leading to the validation of the hypothesis or modification of the model so that the results can verify these assumptions. This part, which is also called validation of the model, is in fact the pure scientific part of the operational research, in general.

After validating the mathematical model, clear conclusions should be drawn to help decision-makers in the organization when the case is. Of these solutions, we have to find the optimal one from the multitude of solutions that exist and this optimal solution must best solve the problem of the hypothesis or the problem of the organization.

Thus, searching for the optimal solution or optimal solution theory is a predilection theme in operational research.

Next, we will try to demonstrate to what a clear definition of the problem we want to solve is being reduced how it is achieved, in contrast to the data acquisition meant to define the hypothesis [2].

Thus, if there are *n* measurable decisions about them, they are represented as decision variables (for example $x_1; x_2 ... x_n$) whose values must be determined. The appropriate measure of performance (e.g. impact) is then expressed as a mathematical function of these decision variables (e.g., I = $3x_1 + 2x_2 + ... + 5x_n$). This function is called the objective function. Any restrictions on the values that can be attributed to these decision variables are also expressed mathematically, usually through inequalities or equations (for example, $x_1 + 3x_1x_2 + 2x \quad \leq 10$). Such mathematical expressions for restrictions are often called constraints. The constants in the formulas that define the **constraints** and the objective function are called the model **parameters**.

The mathematical model should ultimately indicate that the problem is to choose the values of the decision variables to maximize the objective function, subject to the specified constraints.

Such a model and its minor variations are suitable in the study of operational research of the type studied in the management of the organization.

Determining the appropriate values to be attributed to model parameters (one value per parameter) represents both the critical part and the challenge part of the model building process, which will ultimately aim at parameterizing the mathematical model.

Unlike problems in school books, where the mathematical model is parameterized, determining the parameter values for real issues requires collecting relevant data.

Data collection is a fairly difficult part because import and data collection are time-consuming. Therefore, the value assigned to a parameter is often only a gross estimate. Because of the uncertainty about the true value of the parameter, it is important to analyze how the solution deriving from the model would change (or would not change) if the value assigned to the parameter would be modified to other real values.

This process is called sensitivity analysis. Although we refer to the mathematical "model" of a business, for example, a real business normally does not have a "correct" mathematical model. The testing process of a model usually leads to a succession of "fair" models that provide a better and better representation of the problem. It is possible for two or more completely different models to be developed to help analyze the same problem. A particularly important type of mathematical model is the linear programming model, in which the mathematical functions occurring in both the objective function and the constraints are all linear functions [3].

The specific linear programming models are built to suit a variety of issues such as:

- determining the combination of products that maximize profit in hypermarkets;
- effective projection of radiotherapy that fills a tumor, while reducing damage to healthy tissues near the tumor, in medical cases in oncology;
- crop area allocation that maximizes total net profitability and improves agricultural production;
- a combination of pollution reduction methods that achieve air quality standards at a minimal cost in large metropolitan and urban agglomerations.

5. EXAMPLES REGARDING MATHEMATICAL MODEL

5.1 General modeling example

In order to have an overview of a simple mathematical description of a general state system problem, which adequately predicts the response of the physical system to all anticipated inputs, we can describe the system using a normal differential equation as follows:

$$x_1(t), x_2(t), \ldots, x_n(t) \tag{1}$$

With the state variable at time *t*:

$$u_1(t), u_2(t), \ldots, u_m(t) \tag{2}$$

The restrictions of this process, at time *t*, may be described through a differential equation of nth order, as follow:

$$\begin{aligned}
\dot{x}_1(t) &= a_1(x_1(t), x_2(t), \ldots, x_n(t), u_1(t), u_2(t), \ldots, u_m(t), t) \\
\dot{x}_2(t) &= a_2(x_1(t), x_2(t), \ldots, x_n(t), u_1(t), u_2(t), \ldots, u_m(t), t) \\
&\cdot \\
&\cdot \\
&\cdot \\
\dot{x}_n(t) &= a_n(x_1(t), x_2(t), \ldots, x_n(t), u_1(t), u_2(t), \ldots, u_m(t), (t)).
\end{aligned} \tag{3}$$

Thus we will define as state vector:

$$x(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} \tag{4}$$

and control vector:

$$u(t)=\begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix} \tag{5}$$

and so the state equation can be written:

$$x(t)=a(x(t),u(t),t) \tag{6}$$

5.2 Particular modeling example

In other words, if we wanted to model a real economic system using linear programming models to analyze the maximum efficiency of a complex economic system, we can use the structure of the general linear programming model, which is constituted primarily from the set of activities $\{A_1, A_2, ...A_n\}$ that make up the economic system, the resources used $\{R_1, R_2, ...R_m\}$, as well as the multitude of technical and manufacturing relationships.

In order to link activities and resources, we must consider that this is determined by the production technology used by each activity $A_j\ (j=1,...,n)$ being characterized by the column vector $a^{(j)}$ with the components $(a_{1j}, a_{2j}, ..., a_{mj})$.

The elements $\{a_{ij}, i=1,...,m; j=1,...,n\}$ are called technological coefficients or specific technological consumption coefficients and show how much of the resource R_i is consumed to produce a specific product unit P_j, within the activity A_j.

Column vectors or, better known, all manufacturing technologies can be organized in a matrix A with *m* lines and *n* columns where each line refers to an allocated resource $R_i(i=1,...,m)$ and each of the columns refers to an activity $A_j\ (j=1,...,n)$.

Thus, we note with $x_j\ (j=1,...,n)$ the result of the activity A_j in a given period and with $b_i(i=1,...,m)$, the quantities available from the resources $R_i(i=1,...,m)$ and we write mathematically the following technological restrictions, as follows:

$$\begin{array}{l} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \le b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \le b_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \le b_m \end{array} \quad \text{or } A \cdot x \le b \tag{7}$$

where $A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}; x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ și $b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$ (8)

There are two assertions:

1. The amount of resource consumed may not exceed the quantity present at that time;
2. The total consumption R_{ij} of the resource R_i for the achievement of the activity A_j is proportional to the result of the activity, i.e. $x_j, \Rightarrow R_{ij} = a_{ij} \cdot x_j$.

The Restriction System (7) implements the linkage between resources and activities through linear restrictions m.

The linear programming model of a real particular economic system, as outlined above, must contain, in addition to the type (7) restrictions, a performance criterion that allows assessment of the effectiveness of each activity.

Depending on the purpose of modeling, we can choose as an efficiency criterion an indicator that measures the effort or an indicator that measures the outcome [4].

We can also choose as a criterion a ratio between the result and the effort or effort per result, depending on the purpose of the mathematical modeling of the real economic system.

Thus, we have the linear function:

$$f(x) = c_1 x_1 + c_2 x_2 + \dots + c_n x_n \tag{9}$$

which evaluates the efficiency of any real economic system, we obtain the following linear programming program:

$$\underset{x}{optim}\left[f(x)\right] \tag{10}$$

$$\begin{cases} \left.\begin{aligned} \sum_{j=1}^{n} a_{ij} \cdot x_j \le b_1, i \in I_1 \\ \sum_{j=1}^{n} a_{kj} \cdot x_j \ge b_k, k \in I_2 \end{aligned}\right\} \quad I_1 \cup I_2 = \{1,2,\dots,m\} & (11) \\ x_j \ge 0,\ j = \overline{1,n} & (12) \end{cases}$$

Where:

1. Relation (10) is the objective function of the efficiency of the programming problem;
2. Relationship (11) represents resource-type restrictions and qualitative technical-economic restrictions;
3. Relationship (12) is the condition of non-negativity of the variables that actually provide an economically achievable solution.

CONCLUSIONS

Mathematical models have multiple advantages over a verbal description of the problem. One advantage is that a mathematical model describes an issue in a much more concise manner, taking into account internal and external environmental factors. This tends to make the general structure of the problem more understandable and helps to find important cause-effect relationships. In this way, it is more clearly indicated which additional data is relevant for the analysis. It also facilitates solving the problem by taking into account all its interrelationships at the same time. Finally, a mathematical model establishes a link to the use of high-power software-mathematics techniques and computers to analyze the problem.

Indeed, to solve many complex mathematical models, simulation-software, as MATLAB or MAPLE, designed for both personal computers and mainframe computers has become widely available.

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COMPOSITE MODELS USED IN ACTUARIAL PRACTICE

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Abstract: *In this paper, a summary of the composite models developed for use in actuarial practice is presented. We refer in extensive detail to the first composite model introduced in 2005 by Cooray and Ananda [3] which was then generalized by Scollnik [11] in 2007. The main features identified for these models were the density function, the cumulative distribution function, and the n-th order initial moment. We also look into some different variations of these composite models such as: Gamma - Pareto, Weibull - Pareto and Exponential - Pareto models.*

Keywords: *composite model, lognormal distribution, Pareto distribution, Gamma distribution, Weibull distribution, Exponential distribution, parameter estimation.*

1. INTRODUCTION

The modeling of claims data is a major challenge in the construction of applications in general insurance [2]. Insurance companies recorded in time losses that emerge from a combination of moderate and large claims [5]. The modeling of big losses is done in practice with the Pareto distribution. On the other hand, when losses consist of smaller values with high frequencies and larger losses with low frequencies [5], we use the lognormal distribution or Weibull distribution. However in [5], it is underlined that Pareto fits well the tail, but on the other hand, lognormal and Weibull distributions produce an overall good fit but fit badly the tail. Several works have introduced composite models for insurance loss data modeling [1, 6]. Cooray and Ananda [3] in 2005 were the first to open the way for research into composite models using a longnormal distribution to a certain threshold and then the Pareto distribution. Then in 2007 Scollnik [11] generalized the model proposed by Cooray and Ananda proposing two other composite models.

Insurance companies use data on the payment of positive claims. Their distribution often has a high upper tail [3]. Therefore, in the literature, an usual choise is the lognormal distribution or Pareto distribution to model such a data set (see FIG. 1.) [3]. In order to better capture the situations encountered in practice in one model, Cooray and Ananda introduces a composite model that uses lognormal density to a certain threshold and then Pareto density (FIG. 1.). Scollnik (2007) generalizes the proposed composite model [3] and introduces two new composite models (FIG. 2). All these models will be detailed in the next section.

The second section of this article is dedicated to the presentation of the two types of composite models, Cooray and Ananda [3] and Scollnik respectively [11]. For these models, the main features are mentioned: the density function, the cumulative distribution function and the *n-th* order initial moments. It is also presented the drawback of the model proposed by Cooray and Ananda, drawback emphasized by Scollnik in [11].

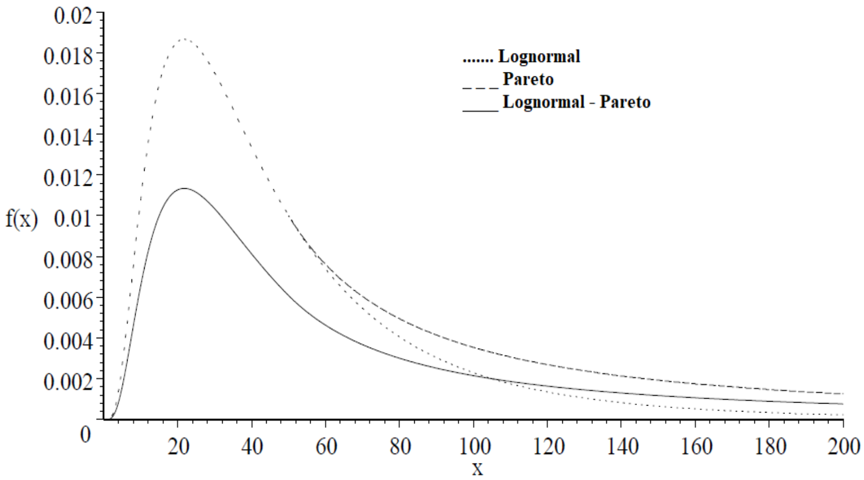

FIG 1. The composite lognormal-Pareto, Cooray and Ananda, model ($\theta = 50, \alpha = 0.5$) [3]

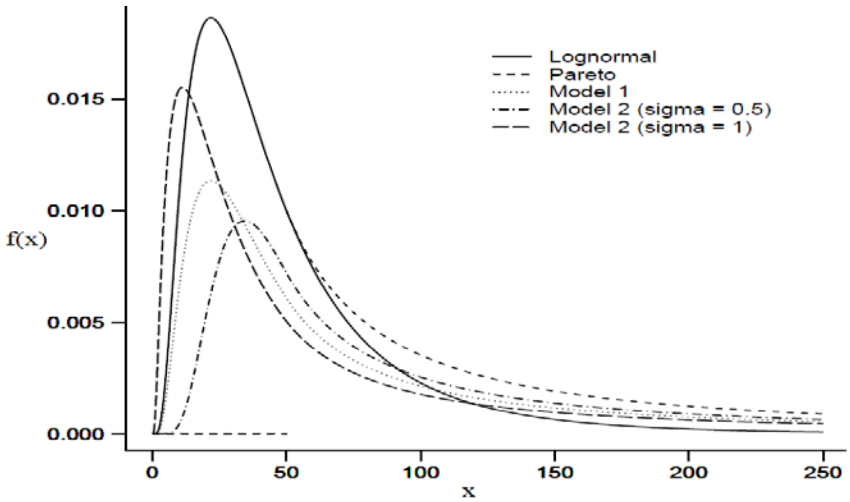

FIG 2. The composite lognormal-Pareto, Scollnik, model ($\theta = 50, \alpha = 0.5$) [11]

In the third section we present the main features of the following particular composite models: Gamma - Pareto, Weibull - Pareto, Exponential - Pareto models. Also, in section four we present a method for the parameter estimation.

2. FIRST COMPOSITE MODELS

2.1. Cooray and Ananda's model

The first lognormal-Pareto composite was developed by Cooray and Ananda (2005) [3] to model insurance payments. Cooray and Ananda construct the composite model considering a random variable X with probability density function:

$$f(x) = \begin{cases} cf_1(x), & 0 < x \leq \theta, \\ cf_2(x), & \theta < x < \infty, \end{cases} \tag{2.1}$$

where $f_1(x)$ and $f_2(x)$ are the lognormal and, respectively, Pareto probability density functions given in (2.2) and (2.3), and c is a normalizing constant.

$$f_1(x) = \frac{(2\pi)^{-1/2}}{x\sigma} \exp\left(-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right), \quad x > 0 \tag{2.2}$$

$$f_2(x) = \frac{\alpha\theta^\alpha}{x^{\alpha+1}}, \qquad x > \theta \tag{2.3}$$

where $\theta, \mu, \sigma, \alpha$ are unknown parameters with the conditions: $\theta > 0, \sigma > 0, \alpha > 0$ and $\mu \in R$.

Imposing the conditions of continuity and differentiability at θ:

$$f(\theta - 0) = f(\theta + 0) \text{ and } f^{'}(\theta - 0) = f^{'}(\theta + 0), \tag{2.4}$$

they rewritten (2.1) in the form:

$$f(x) = \begin{cases} \dfrac{\alpha\theta^\alpha}{\left(1+\Phi(k)\right)x^{\alpha+1}} \exp\left\{-\dfrac{\alpha^2}{2k^2} ln^2\left(\dfrac{x}{\theta}\right)\right\}, & 0 < x \leq \theta, \\ \dfrac{\alpha\theta^\alpha}{\left(1+\Phi(k)\right)x^{\alpha+1}}, & \theta < x < \infty, \end{cases} \tag{2.5}$$

where Φ (.) is the cumulative distribution function of the standard normal distribution. Also the value of the constant k is given by the positive solution of the equation $\exp(-k^2) = 2\pi k^2$. Thus the numerical value obtained for the constant k can be approximated by 0.372238898. Cooray and Ananda (2005) [3] also noticed that $\alpha\sigma = k$ and $c = 1/\left(1 + \Phi(k)\right)$. The conditions imposed in (4) ensure that they have a smooth probability density function. They also reduce the unknown parameters from four to two, $\theta > 0$ and $\alpha > 0$.

In [3] it is shown that the cumulative distribution function of the composite model is:

$$F(x) = \begin{cases} \dfrac{1}{\left(1+\Phi(k)\right)} \Phi\left((\alpha/k)\ln(x/\theta) + k\right), 0 < x \leq \theta, \\ 1 - \dfrac{1}{\left(1+\Phi(k)\right)} (\theta/x)^\alpha, \qquad \theta < x < \infty, \end{cases} \tag{2.6}$$

For the composite lognormal-Pareto model (2.1), Cooray and Ananda show that the *n-th* moment is given by:

$$E(X^n) = \frac{\theta^n}{1+\Phi(k)}\left\{\Phi(k - kn/\alpha)\exp\left[\frac{1}{2}\left(\frac{k}{\alpha}\right)^2(n^2 - 2\alpha n)\right] + \frac{\alpha}{\alpha - n}\right\}, n < \alpha \tag{2.7}$$

Cooray and Ananda [3] show that their proposed model can be applied by actuaries who encounter smaller data with higher frequencies as well as occasionally larger data with lower frequencies. Scollnik in [11] analyzes the model proposed by Corray and Ananda [3] and identifies a significant disadvantage.

Scollnik shows that the model proposed in [3] can be written:

$$f(x)=\begin{cases}\psi\dfrac{1}{\Phi(k)}f_1(x), & 0<x\le\theta,\\ (1-\psi)f_2(x), & \theta<x<\infty,\end{cases} \tag{2.8}$$

where $\theta>0, \alpha>0$ and $f_1(x)$, $f_2(x)$ are the probability density functions given in (2.2) and (2.3). Also, $\Phi(k)=\Phi([\ln(\theta)-\mu]/\sigma)$. It results that:

$$\psi=\frac{\Phi(k)}{1+\Phi(k)}\approx 0.39215 \text{ and } 1-\psi=\frac{1}{1+\Phi(k)}\approx 0.60785 \tag{2.9}$$

In [11] it is shown that the composite model proposed by Cooray and Ananda with fixed and a priori known mixing weights ψ and 1- ψ is restrictive. The theoretical model can be applied to any data set, but in order to obtain an optimal prediction, it is necessary to analyze in advance the set of data from practical activities.

2.2. Scollnik's models

2.2.1. The first composite Scollnik model

Scollnik [11] tried to eliminate the shortcomings of the Cooray and Ananda model by proposing a composite model like a longnormal truncated and Pareto. Thus, he rewroted the probability density function given in (2.1) in the form:

$$f(x)=\begin{cases} rf_1^*(x), & 0<x\le\theta,\\ (1-r)f_2^*(x), & \theta<x<\infty,\end{cases} \tag{2.10}$$

where $0\le r\le 1$, $f_1^*(x)$ and $f_2^*(x)$ represents the truncation of the density function $f_1(x)$ and $f_2(x)$, respectively. Also $f_1(x)$ and $f_2(x)$ are given by (2.2) and (2.3). So we get:

$$\begin{cases} f_1^*(x)=\dfrac{f_1(x)}{F_1(\theta)}, & 0<x\le\theta,\\ f_2^*(x)=\dfrac{f_2(x)}{1-F_2(x)}, & \theta<x<\infty,\end{cases} \tag{2.11}$$

A first observation regarding the composite model proposed by Scollnik is that the value of r is not constant like the value of c in the model proposed by Cooray and Ananda. Here the value of r belongs to the closed interval $[0,1]$ and is dependent on the particular values of θ,μ,σ and α.

In [17], using the continuity of function (2.10) at θ, is obtained:

$$f(\theta-0)=f(\theta+0)\Rightarrow r=\frac{f_2(\theta)F_1(\theta)}{f_2(\theta)F_1(\theta)+f_1(\theta)\big(1-F_2(\theta)\big)}, \tag{2.12}$$

while from the condition of differentiability in θ is obtained:

$$f'(\theta-0)=f'(\theta+0)\Rightarrow r=\frac{f_2'(\theta)F_1(\theta)}{f_2'(\theta)F_1(\theta)+f_1'(\theta)\big(1-F_2(\theta)\big)} \tag{2.13}$$

In [17] the expressions for the cumulative distribution function and for the *n–th* initial moment of the density function (2.10) are calculated. Thus is obtained:

$$F(x) = \begin{cases} r\dfrac{F_1(x)}{F_1(\theta)}, & 0 < x \le \theta, \\ r + (1-r)\dfrac{F_2(x) - F_2(\theta)}{1 - F_2(\theta)}, & \theta < x < \infty, \end{cases} \tag{2.14}$$

and

$$E_n(f) = rE_n(f_1^*) + (1-r)E_n(f_2^*). \tag{2.15}$$

In [13], the advantages of this type of model are presented compared to the non-truncated model.

2.2.2. The second Scollnik model

The second composite model proposed by Scollnik in [11] uses the truncated lognormal distribution for values less than the θ threshold value, and for values greater than the threshold value, uses the generalized Pareto distribution. Thus in [11] it uses the generalized version of Pareto distribution whose density function writes in the form:

$$f_2(x) = \frac{\alpha(\alpha\beta)^\alpha}{(\alpha\beta - \theta + x)^{\alpha+1}}, \qquad x > \theta, \tag{2.16}$$

where $\theta > 0, \alpha > 0$ and $\beta > 0$. If we denote $\gamma = \alpha\beta - \theta$ then the distribution function can be written:

$$f_2(x) = \frac{\alpha(\gamma + \theta)^\alpha}{(\gamma + x)^{\alpha+1}}, \qquad x > \theta, \tag{2.17}$$

where $\theta > 0, \alpha > 0$ and $\gamma > -\theta$. In conclusion, the new composite model has the density function given by the expression:

$$f(x) = \begin{cases} r\dfrac{f_1(x)}{F_1(x)}, & 0 < x < \theta, \\ (1-r)\dfrac{\alpha(\gamma+\theta)^\alpha}{(\gamma + x)^{\alpha+1}}, & \theta < x < \infty, \end{cases} \tag{2.18}$$

where $\theta > 0, \alpha > 0, \gamma > -\theta$ and $r \in [0,1]$. In [17], imposing the conditions of continuity and differentiability at the point θ the expressions for α and *r* were calculated. That's how they got:

$$\alpha + 1 = -\frac{(\gamma + \theta)f_1'(\theta)}{f_1(\theta)}, \tag{2.19}$$

and

$$r = \frac{\alpha f_1'(\theta)F_1(\theta)}{\alpha f_1'(\theta)F_1(\theta) - (\alpha+1)f_1^2(\theta)}, \tag{2.20}$$

Also, in [17], the cumulative distribution function is calculated for this composite model:

$$F(x)=\begin{cases} r\dfrac{F_1(x)}{F_1(\theta)}, & 0<x\leq\theta, \\ 1-(1-r)\left(\dfrac{\gamma+\theta}{\gamma+x}\right)^{\alpha}, & \theta<x<\infty, \end{cases} \tag{2.21}$$

and *n-th* order initial moment:

$$E_n(f)=rE_n(f_1^*)+(1-r)\alpha\sum_{k=0}^{n}\binom{n}{k}\frac{(-\gamma)^{n-k}(\gamma+\theta)^k}{\alpha-k}, \qquad \alpha>n, \tag{2.22}$$

3. FURTHER COMPOSITE MODELS

3.1. Composite Gamma – Pareto models

3.1.1. The first composite Gamma - Pareto model

In [17], the composite model Gamma - Type II Pareto is developed following the construction steps shown in (2.18). For the development of the composite model, we refer to the form of the Gamma function $\Gamma(\nu)=\int_0^\infty x^{\nu-1}e^{-x}dx$ and to $\Gamma(\nu,t)=\int_0^t x^{\nu-1}e^{-x}dx,\ \nu,t>0$, the incomplete Gamma function.

Thus, in [17], the form of the density function for the composite model Gamma-Type II Pareto is given as:

$$f(x)=\begin{cases} r\dfrac{\beta^\delta}{\Gamma(\delta,\beta\theta)}x^{\delta-1}e^{-\beta x}, & 0<x<\theta, \\ (1-r)\dfrac{\alpha(\gamma+\theta)^\alpha}{(\gamma+x)^{\alpha+1}}, & \theta<x<\infty, \end{cases} \tag{3.1}$$

where $\beta,\delta,\alpha,\theta>0,\gamma>-\theta$ and $r\in[0,1]$. Imposing the conditions of continuity and differentiability at the point θ we obtain:

$$\alpha+1=\frac{(\gamma+\theta)(\beta\theta-\delta+1)}{\theta}, \tag{3.2}$$

and

$$r=\frac{\alpha(\beta\theta-\delta+1)\Gamma(\delta,\beta\theta)}{\alpha(\beta\theta-\delta+1)\Gamma(\delta,\beta\theta)+(\alpha+1)(\beta\theta)^\delta e^{-\beta\theta}}, \tag{3.3}$$

Using formula (2.21), in [17], is the form of the cumulative distribution function for this composite model:

$$F(x)=\begin{cases} r\dfrac{\Gamma(\delta,\beta x)}{\Gamma(\delta,\beta\theta)}, & 0<x\leq\theta, \\ 1-(1-r)\left(\dfrac{\gamma+\theta}{\gamma+x}\right)^{\alpha}, & \theta<x<\infty, \end{cases} \tag{3.4}$$

and also using formula (2.22), the *n-th* order initial moment results as:

$$E_n(f) = r\frac{\Gamma(n+\delta,\beta\theta)}{\beta^n\Gamma(\delta,\beta\theta)} + (1-r)\alpha\sum_{k=0}^{n}\binom{n}{k}\frac{(-\gamma)^{n-k}(\gamma+\theta)^k}{\alpha-k},\ \alpha > n, \tag{3.5}$$

In [17], it is underlined that if $\delta = n$ is a positive integer, then the function value $\Gamma(n,.)$ can be written in recursive form:

$$\Gamma(n+1,x) = n\Gamma(n,x) - x^n e^{-x}, \qquad n \geq 1, x > 0, \tag{3.6}$$

with starting value $\Gamma(1,x) = 1 - e^{-x}, x > 0$.

3.1.2. The second composite Gamma – Pareto model

The composite Gamma – Pareto model, developed in [17], results from the composite Gamma – Type II Pareto model for $\gamma = 0$. From this we can obtain the form of the density function for the composite Gamma-Pareto model:

$$f(x) = \begin{cases} r\dfrac{\beta^\delta}{\Gamma(\delta,\beta\theta)}x^{\delta-1}e^{-\beta x}, & 0 < x < \theta, \\ (1-r)\dfrac{\alpha\theta^\alpha}{x^{\alpha+1}}, & \theta < x < \infty, \end{cases} \tag{3.7}$$

where $\alpha > 0, \beta > 0, \delta > 0, \theta > 0$ and $r \in [0,1]$. The parameters α and r were calculated by imposing the conditions of continuity and differentiability for density function (3.7):

$$\alpha = \beta\theta - \delta, \tag{3.8}$$

and

$$r = \frac{(\beta\theta-\delta)\Gamma(\delta,\beta\theta)}{(\beta\theta-\delta)\Gamma(\delta,\beta\theta) + (\beta\theta)^\delta e^{-\beta\theta}}, \tag{3.9}$$

For the model presented in this paragraph, in [17], it is observed that after applying the conditions (3.8) and (3.9) the number of parameters can be reduced from five to three.

If it is attempted to reduce the number of these parameters by using the second derivative, an impossible condition is reached, $\beta\theta^2 = 0$.

The cumulative distribution function, shown in [17], is given by:

$$F(x) = \begin{cases} r\dfrac{\Gamma(\delta,\beta x)}{\Gamma(\delta,\beta\theta)}, & x \leq \theta, \\ 1-(1-r)\left(\dfrac{\theta}{x}\right)^\alpha, & \theta < x < \infty, \end{cases} \tag{3.10}$$

and *n-th* order initial moment is:

$$E_n(f) = r\frac{\Gamma(n+\delta,\beta\theta)}{\beta^n\Gamma(\delta,\beta\theta)} + (1-r)\frac{\alpha\theta^n}{\alpha-n},\ \alpha > n, \tag{3.11}$$

3.2. Composite Weibull – Pareto models

3.2.1. The first composite Weibull – Pareto model

A first composite Weibull-Pareto model was developed by Ciumara (2006). This composite model is built based on the model presented by Cooray and Ananda in (2.1). In [10], a comparative study is made between the two composite distributions, longnormal - Pareto and Weibull - Pareto respectively. In comparison, the density functions, the cumulative distribution functions and the *n-th* order initial moment are discussed. Thus, in (1) they considered:

$$f_1(x) = \frac{\beta}{\gamma^\beta} x^{\beta-1} exp\left(-\left(\frac{x}{\gamma}\right)^\beta\right), x > 0, \gamma > 0, \beta > 1, \tag{3.12}$$

and

$$f_2(x) = \frac{\alpha\theta^\alpha}{x^{\alpha+1}}, x > \theta, \theta > 0, \alpha > 0, \tag{3.13}$$

Thus, in [10], the density function for Weibull-Pareto composite distribution is obtained:

$$f(x) = \begin{cases} \frac{(t_0+1)^2}{(t_0+2)} \frac{\beta}{x} \left(\frac{x}{\theta}\right)^\beta exp\left(-(t_0+1)\left(\frac{x}{\theta}\right)^\beta\right), 0 < x \le \theta, \\ \frac{t_0(t_0+1)}{t_0+2} \frac{\beta}{x} \left(\frac{\theta}{x}\right)^{\beta t_0}, \qquad \theta < x < \infty, \end{cases} \tag{3.14}$$

The value of the constant t_0 was calculated by imposing the conditions of continuity and differentiability of the density function on $(0, \infty)$. This was approximated, in [10], by $t_0 \approx 0.34998$. Also, the number of parameters is reduced from four to two. The other two parameters are expressed using the relationships $\alpha = \beta t_0$ and $\gamma = \theta(t_0+1)^{\frac{1}{\beta}}$. Thus, the constant *c* of the density function definition is, in [10], $c = \frac{t_0+1}{t_0+2}$.

The cumulative distribution function for the Weibull – Pareto composite model is given in [10]:

$$F(x) = \begin{cases} \frac{t_0+1}{t_0+2}\left[1 - exp\left(-(t_0+1)\left(\frac{x}{\theta}\right)^\beta\right)\right], 0 < x \le \theta, \\ 1 - \frac{t_0+1}{t_0+2}\left(\frac{\theta}{x}\right)^{\beta t_0}, \qquad \theta < x < \infty, \end{cases} \tag{3.15}$$

The *n-th* order initial moments are given in [10]:

$$E_n(f) = \frac{t_0+1}{t_0+2}\theta^n\left[(t_0+1)^{-\frac{n}{\beta}}\Gamma\left(\frac{n}{\beta}+1, t_0+1\right) + \frac{\beta t_0}{\beta t_0 - n}\right] \tag{3.16}$$

for $n < \beta t_0$.

3.2.2. The second composite Weibull – Pareto model

The second composite Weibull-Pareto model is the one developed in [14] and [17]. It is a model built on the composite model described in (18). Thus, f_1 represents the Weibull density function, and f_2 represents the Type II Pareto density function.

Under these conditions, the Weibull - Type II Pareto composite model has the density function given by:

$$f(x) = \begin{cases} r\dfrac{1}{1-e^{-(\theta/\tau)^{\beta}}}\dfrac{\beta}{\tau^{\beta}}x^{\beta-1}e^{-(x/\tau)^{\beta}}, & 0 < x < \theta, \\ (1-r)\dfrac{\alpha(\gamma+\theta)^{\alpha}}{(\gamma+x)^{\alpha+1}}, & \theta < x < \infty, \end{cases} \tag{3.17}$$

where the parameters $\beta, \tau, \alpha, \theta > 0, \gamma > -\theta$ and $r \in [0,1]$. Imposing the conditions of continuity and differentiability on $(0, \infty)$, in [17], it is shown that:

$$\alpha + 1 = \frac{\gamma+\theta}{\theta}\left[\beta(\theta/\tau)^{\beta} - \beta + 1\right], \tag{3.18}$$

and

$$r = \frac{\alpha\left[\beta(\theta/\tau)^{\beta} - \beta + 1\right]\left[e^{(\theta/\tau)^{\beta}} - 1\right]}{\alpha\left[\beta(\theta/\tau)^{\beta} - \beta + 1\right]\left[e^{(\theta/\tau)^{\beta}} - 1\right] + (\alpha+1)\beta(\theta/\tau)^{\beta}}, \tag{3.19}$$

Thus, the number of unknown parameters has been reduced from six to four. Also, in [17], it is shown that if it is still desired to reduce the number of unknown parameters the condition obtained is:

$$\begin{aligned} r\beta(\theta/\tau)^{\beta}\,\frac{(\beta-1)(\beta-2) - 3\beta(\beta-1)(\theta/\tau)^{\beta} + \beta^{2}(\theta/\tau)^{2\beta}}{\theta^{3}\left[e^{(\theta/\tau)^{\beta}} - 1\right]} = \\ = (1-r)\alpha(\alpha+1)(\alpha+2)\frac{1}{(\gamma+\theta)^{3}}, \end{aligned} \tag{3.20}$$

where $\gamma = \frac{\beta^{2}\theta^{\beta+1}}{(1-\beta)(\tau^{\beta}+\beta\theta^{\beta})}$.

The cumulative distribution function for the Weibull-Type II Pareto composite model in [17] is:

$$F(x) = \begin{cases} r\dfrac{1-e^{-(x/\tau)^{\beta}}}{1-e^{-(\theta/\tau)^{\beta}}}, & 0 < x \le \theta, \\ 1-(1-r)\left(\dfrac{\gamma+\theta}{\gamma+x}\right)^{\alpha}, & \theta < x < \infty, \end{cases} \tag{3.21}$$

Here it was used that $F_1(x) = 1 - e^{-(x/\tau)^{\beta}}$.

For the composite Weibull - Type II Pareto the *n-th* order initial moment is given in [17]:

$$E_n(f) = r\frac{\tau^n}{1-e^{-(\theta/\tau)^\beta}}\Gamma\left(\frac{n}{\beta}+1,(\theta/\tau)^\beta\right) + (1-r)\alpha\sum_{k=0}^{n}\binom{n}{k}\frac{(-\gamma)^{n-k}(\gamma+\theta)^k}{\alpha-k}, \qquad \alpha > n, \tag{3.22}$$

3.2.3. The third composite Weibull – Pareto model

The third composite Weibull - Pareto model, developed in [17], represents a particular case of the composite Weibull - Type II Pareto model, for $\gamma = 0$. This gives the density function for the composite Weibull-Pareto model:

$$f(x) = \begin{cases} r\frac{1}{1-e^{-(\theta/\tau)^\beta}}\frac{\beta}{\tau^\beta}x^{\beta-1}e^{-(x/\tau)^\beta}, & 0 < x < \theta, \\ (1-r)\frac{\alpha\theta^\alpha}{x^{\alpha+1}}, & \theta < x < \infty, \end{cases} \tag{3.23}$$

where $\alpha, \beta, \tau, \theta > 0$ and $r \in [0,1]$. After applying the conditions of continuity and differentiability of the density function (3.23), it is obtained in [17]:

$$\alpha = \beta(\theta/\tau)^\beta - \beta + 1, \tag{3.24}$$

and

$$r = \frac{\alpha\left[e^{(\theta/\tau)^\beta} - 1\right]}{\alpha\left[e^{(\theta/\tau)^\beta} - 1\right] + \beta(\theta/\tau)^\beta}, \tag{3.25}$$

And this time, due to the application of conditions (3.26) and (3.27), the reduction of unknown parameters is from five to three. In [17], the observation is made that if it is attempted to reduce the number of unknown parameters to two it follows that $\beta^2\theta^{\beta+1} = 0$, which leads to $\beta = 0$, which is impossible, or $\theta = 0$ in which case the proposed model turns into the classic Pareto.

Also, in [17], we get the expressions for the cumulative distribution function:

$$F(x) = \begin{cases} r\frac{1-e^{-(x/\tau)^\beta}}{1-e^{-(\theta/\tau)^\beta}}, & 0 < x \le \theta, \\ 1-(1-r)\left(\frac{\theta}{x}\right)^\alpha, & \theta < x < \infty, \end{cases} \tag{3.26}$$

and *n-th* order initial moment for the composit Weibull – Pareto model:

$$E_n(f) = r\frac{\tau^n}{1-e^{-(\theta/\tau)^\beta}}\Gamma\left(\frac{n}{\beta}+1,(\theta/\tau)^\beta\right) + (1-r)\frac{\alpha\theta^n}{\alpha-n}, \ \alpha > n, \tag{3.27}$$

3.3. Composite Exponential – Pareto models

3.3.1. The first composite Exponential – Pareto model

The composite exponential - Pareto model was developed in [16] according to the model described in [3]. Thus, in the model constructed in (2.1), f_1 is considered to be the exponential density and f_2 the Pareto density. In conclusion, in [16], it is considered:

$$f_1(x) = \lambda e^{-\lambda x}, x > 0, \tag{3.28}$$

$$f_2(x) = \frac{\alpha\theta^{\alpha}}{x^{\alpha+1}}, x > \theta, \tag{3.29}$$

where $\lambda > 0, \alpha > 0, \theta > 0$ are unknown parameters.

Following the application of the constraints of continuity and differentiability of the density function it is obtained in [16]:

$$\begin{cases} \lambda e^{-\lambda\theta} = \dfrac{\alpha}{\theta}, \\ \lambda^2 e^{-\lambda\theta} = \dfrac{\alpha(\alpha+1)}{\theta^2}, \end{cases} \tag{3.30}$$

The authors get:

$$\begin{cases} \lambda\theta = 1.35 \\ \alpha = 0.35 \end{cases} \tag{3.31}$$

After the restriction system was resolved, it was possible to reduce the unknown parameters from three to one. Also, imposing the condition $\int_0^{\infty} f(x)dx = 1$ the normalization constant is obtained:

$$c = \frac{1}{2 - e^{-\lambda\theta}} = 0.574, \tag{3.32}$$

The density function for the composite Exponential - Pareto model can be written as:

$$f(x) = \begin{cases} \dfrac{0.775}{\theta} e^{-\frac{1.35x}{\theta}}, & 0 < x \le \theta, \\ 0.2 \dfrac{\theta^{0.35}}{x^{1.35}}, & \theta < x < \infty, \end{cases} \tag{3.33}$$

And the cumulative distribution function, in [16], is given as:

$$F(x) = \begin{cases} 0.574\left(1 - e^{-\frac{1.35x}{\theta}}\right), 0 < x \le \theta, \\ 1 - 0.574\left(\dfrac{\theta}{x}\right)^{0.35}, \theta < x < \infty, \end{cases} \tag{3.34}$$

3.3.2. The second composite Exponential – Pareto model

The second composite Exponential – Pareto model, developed in [15], is built on the model (2.18). A generalized Pareto distribution is used, in [15], above the threshold θ. Thus, the second Exponential – Pareto composite model has the density function:

$$f(x) = \begin{cases} r \dfrac{\lambda e^{-\lambda x}}{1 - e^{-\lambda\theta}}, & 0 < x \le \theta, \\ (1-r)\dfrac{\alpha(\gamma+\theta)^{\alpha}}{(\gamma+x)^{\alpha+1}}, \theta < x < \infty, \end{cases} \tag{3.35}$$

where the parameters $\lambda, \alpha, \theta > 0, \gamma > -\theta$ and $r \in [0,1]$. In [15], applying the conditions of continuity and differentiability on $(0, \infty)$, the following conditions are obtained:

$$\alpha + 1 = \lambda(\gamma + \theta), \tag{3.36}$$

and

$$r = \frac{\alpha\left(1 - e^{-\lambda\theta}\right)}{\alpha + e^{-\lambda\theta}}, \tag{3.37}$$

It is noted that following the application of the two conditions, continuity and differentiability, the number of unknown parameters decreased from five to three. In [15] trying to reduce the number of unknown parameters using a second derivative requirement yields:

$$\frac{r}{1 - e^{-\lambda\theta}} \lambda^3 e^{-\lambda\theta} = (1 - r)\alpha(\alpha + 1)(\alpha + 2) \frac{1}{(\gamma + \theta)^3}, \tag{3.38}$$

and using (3.36) and (3.37) the authors conclude $\alpha + 1 = \alpha + 2 \Leftrightarrow 0 = 1$, which is impossible.

The cumulative distribution function for the composite Exponential - Type II Pareto model is:

$$F(x) = \begin{cases} r\dfrac{1 - e^{-\lambda x}}{1 - e^{-\lambda x}}, 0 < x \leq \theta, \\ 1 - (1 - r)\left(\dfrac{\gamma + \theta}{\gamma + x}\right)^{\alpha}, \theta < x < \infty, \end{cases} \tag{3.39}$$

and the *n-th* order moment of the composite Exponential – Type II Pareto is given by, in [15]:

$$E_n(f) = r\frac{\Gamma(n + 1, \lambda\theta)}{\lambda^n(1 - e^{-\lambda\theta})} + (1 - r)\alpha \sum_{k=0}^{n} \binom{n}{k} \frac{(-\gamma)^{n-k}(\gamma + \theta)^k}{\alpha - k}, \alpha > n, \tag{3.40}$$

where Γ is the incomplete gamma function.

3.3.3. The third composite Exponential – Pareto model

The third Exponential - Pareto composite model, developed in [15], is designed according to the model (2.18). It's a composite Exponential-Pareto model define as a truncated Exponential and Pareto mixture with threshold value θ [15]. In other words, this model is a particular case obtained by taking, $\gamma = 0$, of the composite model presented in the previous section.

The third composite Exponential – Pareto model has the density function:

$$f(x) = \begin{cases} r\dfrac{\lambda e^{-\lambda x}}{1-e^{-\lambda\theta}}, & 0 < x \le \theta, \\ (1-r)\dfrac{\alpha\theta^{\alpha}}{x^{\alpha+1}}, & \theta < x < \infty, \end{cases} \tag{3.41}$$

where $\lambda > 0, \alpha > 0, \theta > 0$ and $r \in [0,1]$. Also, by imposing the conditions of continuity and differentiability on $(0, \infty)$ to the density function (3.41), one obtains as in [15]:

$$\alpha + 1 = \lambda\theta, \tag{3.42}$$

and

$$r = \frac{\alpha\left(1-e^{-\lambda\theta}\right)}{\alpha + e^{-\lambda\theta}} = \frac{\alpha\left(1-e^{-(\alpha+1)}\right)}{\alpha + e^{-(\alpha+1)}}, \tag{3.43}$$

In [15], it is shown that because of conditions (3.42) and (3.43) the number of unknown parameters is reduced from four to two. If it is still desired to reduce the number of unknown parameters one can use the second order derivative. This leads to:

$$\frac{r}{1-e^{-\lambda\theta}}\lambda^3 e^{-\lambda\theta} = (1-r)\alpha(\alpha+1)(\alpha+2)\frac{1}{\theta^3} \tag{3.44}$$

But if in relation (3.44) we use relations (3.42) and (3.43) we are led to $\alpha + 1 = \alpha + 2 \Leftrightarrow 0 = 1$, which is impossible.

The cumulative distribution function for the third composite Exponential - Pareto is given in [15]:

$$F(x) = \begin{cases} r\dfrac{1-e^{-\lambda x}}{1-e^{-\lambda x}}, & 0 < x \le \theta, \\ 1-(1-r)\left(\dfrac{\theta}{x}\right)^{\alpha}, & \theta < x < \infty, \end{cases} \tag{3.45}$$

And the initial *n-th* order moments are:

$$E_n(f) = r\frac{\Gamma(n+1,\lambda\theta)}{\lambda^n\left(1-e^{-\lambda\theta}\right)} + (1-r)\frac{\alpha\theta^n}{\alpha-n}, \alpha > n, \tag{3.46}$$

where Γ is the incomplete gamma function.

4. PARAMETER ESTIMATION

An important aspect is the necessity of estimating the unknown parameter θ. Many studies [7,8] use on the maximum likelihood method as the method of estimating parameter. Thus, consider the case of the composite model proposed by Scollnik (2007) whose density function is given by (2.18), with the real parameters $\delta_1, \delta_2, \delta_3, \dots, \delta_s, \theta$, with $s \in N$ and $x_1 \le x_2 \le x_3 \le \cdots \le x_n$ an ordered sample of data from the composite (2.18). In [16], it is specified that if the likelihood function is to be evaluated, it is necessary to know where the unknown parameter θ is placed in relation to that sample.

Assuming that the unknown parameter θ is placed $x_m \leq \theta \leq x_{m+1}$, then the likelihood function is given in [17]:

$$L(x_1, \dots, x_n, \delta_1, \dots \delta_n, \theta) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{m} r f_1^*(x_i) \prod_{i=m+1}^{n} (1-r) f_2^*(x_j)$$
$$= r^m (1-r)^{n-m} \prod_{i=1}^{m} f_1^*(x_i) \prod_{i=m+1}^{n} f_2^*(x_j). \tag{4.1}$$

Since the proposed method depends on m, which is not known exactly, in [17] the following algorithm is proposed:

Step 1. For each $m = 1,2,3, \dots, n-1$, evaluate $\widehat{\delta_1}, \dots, \widehat{\delta}_s, \hat{\theta}$ as solutions of the system:

$$\begin{cases} \dfrac{\partial \ln L}{\partial \delta_i} = 0, \quad i = 1,2,3, \dots, s, \\ \dfrac{\partial \ln L}{\partial \theta} = 0 \end{cases} \tag{4.2}$$

If $\hat{\theta}$ is located between $x_m \leq \hat{\theta} \leq x_{m+1}$ then $\hat{\theta}$ is the maximul likelihood estimator. If not, go to step 2

Step 2. If the system (4.2) has no solution then we are in one of two situations $m = n$ or $m = 0$. In this case, in [17] it is recommended to use one of the functions f_1 or f_2 for the likelihood function.

Unknown parameter estimation using this method can be implemented on a computing platform as shown in [12]. This may lead to reduced work-time in terms of system solving (4.1).

CONCLUSIONS

In this paper we presented a summary of the characteristics of the main composite models used in the processing of statistical data in actuarial. We begin the work with the basic composite models, Cooray and Ananda (2005) and Scollnik (2007), and then introduce the particular Gamma-Pareto, Weibull-Pareto and Exponential-Pareto models. Depending on the distribution of the data to be processed, one or other of the models presented may be applied.

There are other composite models studied in the literature like, e.g.: composite truncation models [13], composite lognormal – Burr [1], composite Stoppa models [2], inverse Weibull composite models [4], composite lognormal – Pareto model with random threshold [9].

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