A FAST SELF-ADAPTIVE APPROACH TO RELIABILITY OPTIMIZATION PROBLEMS

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Abstract: *The paper is investigating the suitability of the FSA-DE optimization method for solving reliability optimization problems by approaching a set of three case studies: a known RAP case study, a FTO case study and a ETO case study. For the RAP case study the numerical results obtained by FSA-DE are compared with the ones obtained by other known optimization methods.*

Keywords: *Mixed Integer Non-Linear Programming (MINLP), Global Optimization (GO), Fast Self-Adaptive Differential Evolution (FSA-DE), Reliability Optimization, Redundancy Allocation Problem (RAP), Fault Tree Optimization (FTO), Event Tree Optimization (ETO)*

1. INTRODUCTION

In order to become more competitive on the market, many manufacturers are investing resources for improving the reliability of the systems and components they produce. Two approaches are commonly used in order to reach a high reliability of a system.

In the first approach the system's reliability is increased during the design phase by increasing the number of redundant components in the various subsystems of the considered system. But by increasing the number of identical components there are also involved increases in the cost, the weight or the volume of the sub-systems, which impose additional constraints on the overall cost, weight or volume of the system. This first model is called the Redundancy Allocation Problem (*RAP*) and was first introduced by Fyffe et al. in [1]. There are many varieties of RAP problems in the field of reliability optimization, which were widely investigated by using many optimization methods, including the meta-heuristic ones. For an overview of *RAP* see Kuo and Prasad [2], and for surveys of the most recent research advances in *RAP* problems see Kuo [3] and Chambari et al. [4].

In the second approach the system reliability is increased by increasing the reliability of the components, and it can be applied to both the design and operational phases of the system. In order to determine the components which should be considered for reliability improvement and their optimal reliability values, taking into account that there are also some economic cost limitations, as opposed to the reliability (safety) requirements, some combinations of Fault Tree Analysis (*FTA*) (see [5]) techniques and mathematical optimization techniques are employed. This second model is called the Fault Tree Optimization (*FTO*) problem and it was investigated by applying mainly Genetic Algorithms (*GA*) optimization techniques (see [6], [7]).

The models and methodologies based on probabilistic risk analysis and optimization can be extended from optimizing the design and operation of systems and sub-systems to optimizing the design and operation of complex industrial systems, like nuclear power plants, or fossil power plants (see [8]).

The goal of such a methodology is to minimize the risk to have a nuclear accident or the economic risk to shut down the production for all the possible reasons.

In design the focus is on component quality and redundancy levels, while in maintenance and testing the focus is on scheduling tasks and human reliability. After modeling the systems and sub-systems by using the *FTA* methodology, the next step is to model the Accidental Sequences (*AS*) with Event Trees (*ET*) ([9]). In an *AS* several systems are performing their functions successfully or unsuccessfully. The combination of different systems performing their functions right or wrong drive the *AS* to different final Plant States (*PS*) which can be grouped, according to the degree of damage produced, as totally successful, partially successful, or unsuccessful. The plant states can be quantified and some constraints can be imposed on the unsuccessful states according to some permissible upper and lower risk limits. When the total investment and the operating budget is limited, the Event Tree Optimization (*ETO*) problem consists in how to optimally distribute the funds so that all the unsuccessful Plant States in *ET* are observing the imposed permissible risk limits.

2. MIXED INTEGER NON-LINEAR PROGRAMMING (*MINLP*) PROBLEM

The most general form of the reliability optimization problems treated in this paper is the *MINLP* formulation where equality or inequality constraints can be applied to the objective function and some of the decision variables can take continuous real values in real intervals, while other decision variables are restricted to integer values in sets of consecutive integer values ([10]):

$$\begin{array}{ll} \text{minimize} & f(\mathbf{x}) \\ \text{subject to} & \mathbf{x} \in \mathrm{D} \end{array} \qquad (1)$$

with:

$$\mathrm{D} = \left\{ \mathbf{x}:\ \mathbf{l} \le \mathbf{x} \le \mathbf{u};\ \text{and}\ g_i(\mathbf{x}) \le 0, i = 1,\dots,\mathrm{G};\ \text{and}\ h_j(\mathbf{x}) = 0, j = 1,\dots,\mathrm{H} \right\} \quad (2)$$

where, $\mathbf{x} \in \mathbb{R}^n$ is a real n-dimensional vector of decision variables ($\mathbf{x} = (x_1, x_2, \dots, x_n)$), there is a number $0 \le n_C \le n$ such that the last $n - n_C$ decision variables are restricted to integer values, $f: \mathbb{R}^n \to \mathbb{R}$ is the continuous objective function, $\mathrm{D} \subset \mathbb{R}^n$ is the non-empty set of feasible decisions (a proper subset of $\mathbb{R}^n$), $\mathbf{l}$ and $\mathbf{u}$ are explicit, finite (component-wise) lower and upper bounds of $\mathbf{x}$, $g_i: \mathbb{R}^n \to \mathbb{R}$, $i = 1,\dots,\mathrm{G}$ is a finite collection of continuous inequality constraint functions, and $h_j: \mathbb{R}^n \to \mathbb{R}$, $j = 1,\dots,\mathrm{H}$ is a finite collection of continuous equality constraint functions. No other additional suppositions are made on the *MINLP* problem and it is assumed that no additional knowledge about the objective function and constraint functions can be obtained, in this way treating the *MINLP* problem as a black box, i.e. for any point $\mathbf{x}$ in the boxed domain $\{\mathbf{x} : \mathbf{l} \le \mathbf{x} \le \mathbf{u}\}$ it is assumed the ability to calculate the values of the functions $f(\mathbf{x})$, $g_i(\mathbf{x}), i = 1,\dots,\mathrm{G}$, $h_j(\mathbf{x}), j = 1,\dots,\mathrm{H}$, but nothing more. In order to efficiently handle the constraints in constrained optimization problems one of the best approaches is to apply the Deb's Rules (see [11]). For a detailed constraints handling methodology based on Deb's Rules see [12]. In the *MINLP* model another important issue is the handling of the integer constraints. In the population based meta-heuristic optimization methods the integer decision variables are treated like the continuous variables, but when the objective function $f(\mathbf{x})$ is evaluated the values rounded to the closest integer, $x'_j = round(x_j), j = n_C + 1,\dots,n$, are used in the evaluation.

In [12] the *FSA-DE* variant of Differential Evolution (*DE)* was constructed by implementing and experimentally testing a set of four gradual and cumulative improvements to the initial *DE/rand/1/bin* scheme (originally introduced in [13]): 1) a randomization of the scaling control parameter in the real interval $[0,1)$, 2) a Random Greedy Selection method (*RGS*, see[14]);

3) the use of a normal (Gaussian) probability distribution for sampling the crossover probability, and 4) a resetting mechanism. *FSA-DE* proved better performance, while the dependence on method parameters was eliminated.

3. RAP CASE STUDY

We consider a known *RAP* case study, the 5-unit bridge structure shown in **FIG. 1** (see [15]):

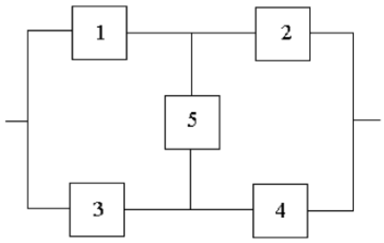

FIG. 1. The schematic diagram of 5-unit bridge system ([15])

In order to efficiently compute the reliability of this system we first eliminate the bridge components by applying the Bayes Total Probability Theorem:

$$R_S(\mathbf{r},\mathbf{n}) = R(s_5)R(S|s_5) + R(\bar{s}_5)R(S|\bar{s}_5) = \\ = R_5R(S_1) + (1 - R_5)R(S_2) \quad (3)$$

where we used the notations S for the original bridge system, $s_1, s_2, \ldots, s_5$ for the boolean states of the sub-systems (*true* means perfectly working, while *false* means totally defective) and $R_i = R(s_i),\ i = 1,\ldots,5$ are the reliabilities of the sub-systems. The simplified systems $R(S_1)$ and $R(S_2)$ are of serial-parallel type and their reliabilities can be easily calculated: $R(S_1) = (1-(1-R_1)(1-R_3))(1-(1-R_2)(1-R_4))$ and $R(S_2) = 1-(1-R_1R_2)(1-R_3R_4)$. The sub-systems are comprising identical redundant (parallel connected) components, and their reliabilities are simply calculated as $R_i = 1-(1-r_i)^{n_i},\ i = 1,\ldots,5$, with n_i the number of components in subsystem i. The optimization problem for the 5-unit has the dimension 10, the decision variables being the numbers of components n_i and the component reliabilities r_i:

maximize $\quad R_S(\mathbf{r},\mathbf{n})$

subject to $\quad C_S(\mathbf{r},\mathbf{n}) \le C_{lim},\ W_S(\mathbf{r},\mathbf{n}) \le W_{lim},\ V_S(\mathbf{r},\mathbf{n}) \le V_{lim} \quad (4)$

where: $C_S(\mathbf{r},\mathbf{n}) = \sum_{i=1}^{5} \alpha_i \left(-\frac{T}{\ln(r_i)}\right)^{\beta_i} (n_i + e^{n_i/4})$, $W_S(\mathbf{r},\mathbf{n}) = \sum_{i=1}^{5} w_i n_i e^{n_i/4}$ and $V_S(\mathbf{r},\mathbf{n}) = \sum_{i=1}^{5} w_i v_i {n_i}^2$. The other constraints applied to the 5-unit bridge system are: $1 \le n_i \le 10, 0.0 \le r_i \le 1.0,\ i = 1,\ldots,5$, with n_i taking integer values and r_i taking real values. The design data are $C_{lim} = 175.0$, $W_{lim} = 200.0$, $V_{lim} = 110.0$ and $T = 1000h$. The other design constants are the ones given in [15]. **Table 1** gives the best results obtained by *FSA-DE* method for 5-unit bridge system problem.

Table 1 – *RAP* Case Study, 5-Unit Bridge System, *FSA-DE* results

Objective	Stage	r_i	n_i	Attribute
$Max\ R_S$	1	0.828045581	3	$R_S = 0.99988964$
	2	0.857778608	3	$C_S = 74.9994$
	3	0.914351326	2	$W_S = 198.439534$
	4	0.648110393	4	$V_S = 105$
	5	0.704001133	1	

Table 2 gives some comparison results between various other recently employed optimization methods and *FSA-DE* method for the 5-unit bridge system case study, and it can be observed that *FSA-DE* method was able to achieve the best known maximum reliability.

Table 2 - *RAP* Case Study, 5-Unit Bridge System, comparative results

Parameter	*HS* [16]	*IPSO* [17]	*ABC* [18]	*ICS* [15]	*FSA-DE*
$R_S(\mathbf{r}, \mathbf{n})$	0.99988962	0.99988963	0.99988962	0.99988964	**0.99988964**
n_1	3	3	3	3	**3**
n_2	3	3	3	3	**3**
n_3	2	2	2	2	**2**
n_4	4	4	4	4	**4**
n_5	1	1	1	1	**1**
r_1	0.82883148	0.82868361	0.828087	0.828094038	**0.828045581**
r_2	0.85836789	0.85802567	0.857805	0.858004485	**0.857778608**
r_3	0.91334996	0.91364616	0.914240	0.914162924	**0.914351326**
r_4	0.64779451	0.64803407	0.648146	0.647907792	**0.648110393**
r_5	0.70178737	0.70227595	0.704163	0.704565982	**0.704001133**

6. FTO CASE STUDY

The first step in a *FTA* methodology involves the construction of the fault tree representation of the system. Usually a top-down approach is adopted, starting from the definition of the general failure condition of the system (the top event) and logically developing the fault tree structure, through *OR*, *AND* and *NOT* logical gates, from more general failure events to more specific failure events associated to the sub-systems and the components of a system. When the logic cannot be further developed, the last generated events are considered the basic events of the fault tree. Each basic event has an associated reliability r_i and the corresponding amount of investment c_i which is needed to achieve the reliability. For each basic event it is needed a reliability-cost curve, which is available from mathematical modeling and calculation, historical data, or can be provided by the manufacturer. A typical reliability-cost curve is presented in **FIG. 2**, where it can be observed that the reliability is increasing with the cost and it is asymptotically approaching the value of 1.0 with a very high cost. When the costs are known, once calculated the reliabilities of the basic events $r_i(c_i)$, the overall reliability of the system can be calculated using the fault tree logic. We can define the following optimization problem for Reliability Maximization (achieving the maximum possible system reliability within a given amount of investment):

$$\text{maximize} \quad R_{TOP}(\mathbf{c})$$

$$\text{subject to} \quad \sum_{i=1}^{N_{be}} c_i \leq C_{tot} \qquad (5)$$

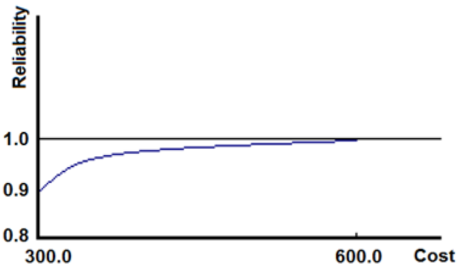

FIG. 2. A typical reliability-cost curve ([6])

where N_{be} is the number of basic events, $c_i, i = 1, \ldots, N_{be}$ are the associated costs and C_{tot} is the total investment. We built a simple *FTO* case study starting from a simple fault tree with seven basic events (see [19]) presented in **FIG. 3**.

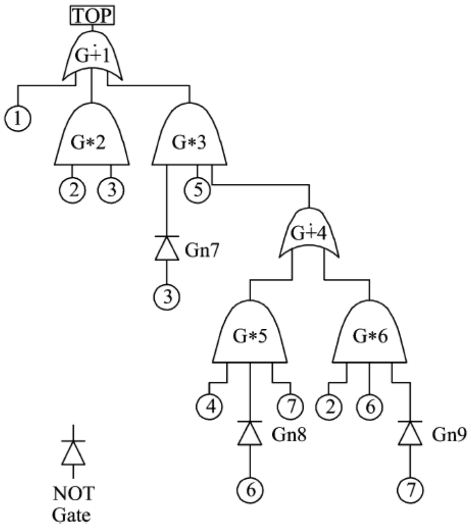

FIG. 3. A fault tree example ([19])

We can logically evaluate the *TOP* event by applying a top-down approach: $TOP = e_1 + G_2 + G_3 = e_1 + e_2 e_3 + \bar{e}_3 e_5 G_4$, with $G_4 = G_4 + G_4 = e_4 \bar{e}_6 e_7 + e_2 e_6 \bar{e}_7$. In order to evaluate the system reliability we apply recursively the Bayes Total Probability Theorem by first eliminating the basic events which appear multiple times, and after that evaluating the remaining tree structures by applying a simple bottom up approach. We have:

$$R_{TOP} = R(TOP) = 1 - Q(TOP) \tag{6}$$

$$\begin{aligned} Q(TOP) &= q_3 Q(TOP|e_3) + r_3 Q(TOP|\bar{e}_3) = \\ &= q_3(1 - r_1 r_2) + r_3 \left(1 - r_1\left(1 - q_5 Q(G_4)\right)\right) \end{aligned} \tag{7}$$

with

$$Q(G_4) = q_6 Q(G_4|e_6) + r_6 Q(G_4|\bar{e}_6) = q_6 q_2 r_7 + r_6 q_4 q_7 \tag{8}$$

We modeled the reliability-cost curve by simply using the hyperbolic tangent function:

$$r_i(c) = \frac{e^{\alpha_i c} - 1}{e^{\alpha_i c} + 1} \tag{9}$$

with the constants $\alpha_i > 0,\ i = 1, \ldots, 7$ (which control the slope of the curve) given in **Table 3**.

We assumed that we started from an initial investment of 7000 units which is distributed among the basic events, so that with the minimal costs $c_{min,i}, i = 1, \dots, 7$ given in **Table 3**, the unreliabilities $q0_i, i = 1, \dots, 7$ are the same as given in [19].

Table 3 - Data for *FTO* case study

i	α_i	$q0_i$	$c_{min,i}$
1	0.003701	0.01614	1300.0
2	0.004292	0.0625	800.0
3	0.004215	0.3125	400.0
4	0.003701	0.01614	1300.0
5	0.002837	0.00125	2600.0
6	0.003662	0.5	300.0
7	0.003662	0.5	300.0

The available investment was $C_{tot} = 12000$ units, with an additional investment of 5000 units. **Table 4** gives the best result obtained by *FSA-DE* method. It can be observed that with an additional investment of 5000 units distributed among the basic events e_1, e_2 and e_4 the system's reliability was increased from 0.96461053 (which is obtained with the minimal investment of 7000 units) to 0.99999577.

Table 4 - *FTO* case study, *FSA-DE* results

c_1	c_2	c_3	c_4	c_5	c_6	c_7	$R_{TOP}(\mathbf{c})$
3815.276	3053.641	400.0	1531.068	2600.0	300.0	300.0	0.99999577

7. ETO CASE STUDY

The *ETO* case study considered in this section (see **FIG. 4**.) is based on a simplified Event Tree obtained from the original Event Tree built for CAREM 25 Project ([20]). CAREM 25 is a CNEA (Comisión Nacional de Energía Atómica) project from Argentina aiming to develop, design and construct a small nuclear power plant with an electrical output of about 27 MW. According to [20], the Accidental Sequences (*AS*) were built simplifying the headers to show only the human error intervention. Five models were taken into account for the representation of the human behavior: 1) Technician, 2) Technician and supervision, 3) Technician and supervision with written procedures, 4) Technician and administrative control, 5) Technician, supervision and administrative control.

To each human behavior model a human error probability (or error frequency) $\boldsymbol{q_j}, \boldsymbol{j} = \mathbf{1}, \dots, \mathbf{5}$, was associated, as determined in [20] by applying Human Event Tree (*HEP*) modeling. Also a cost, $\boldsymbol{c_j}, \boldsymbol{j} = \mathbf{1}, \dots, \mathbf{5}$, was associated, the data being presented in **Table 5**. The *AS*s in an *ET* are initiated at the left by an undesired event ue, which in the considered case study also came from a human error. The ASs are simplified in order to show in the headers only the human interventions : $\boldsymbol{he_1}, \boldsymbol{he_2}, \boldsymbol{he_3}, \boldsymbol{he_4}$. An up branch is representing a successful intervention and an down branch is representing a wrong intervention. On the right hand side the final Plant States (*PS*) are represented.

The successful *PS*s and the successful human interventions are represented underlined. For an unsuccessful *PS* a frequency between 10^{-7} and 10^{-9} is considered a reasonable value. Any value lower than 10^{-9} represents a too high cost and any value higher than 10^{-7} represents a very high risk.

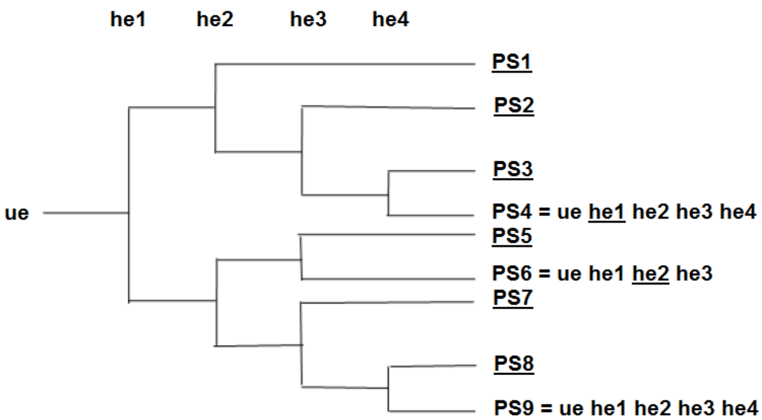

FIG. 4. The simplified CAREM 25 Event Tree ([20])

Table 5 - Data for *ETO* case study

Model	1	2	3	4	5
error freq.	0.505	0.1009	0.001105	0.0209	0.0011049
Cost	240.0	520.0	1530.0	950.0	1670.0

We are required to solve here a combinatorial type of optimization problem where we need to associate to each human intervention $he_i, i = 1, \dots, 4$ in the *ET* an appropriate model $m_i, i = 1, \dots, 4$, from the five given models, so that the total cost associated to the human intervention is minimal, while the frequencies of the unsuccessful *PS*s are observing the required constraints. This problem can be easily modeled as an integer programming problem by taking as decision variables the indices of the selected human behavior models:

$$\text{minimize} \qquad C_{tot} = \sum_{i=1}^{4} C(m_i)$$

$$\text{subject to} \qquad 10^{-9} \le Q(PS_4), Q(PS_6), Q(PS_9) \le 10^{-7} \qquad (10)$$

where $Q(PS_4) = Q(ue\ \overline{he}_1 he_2 he_3 he_4) = Q(ue)(1 - Q(m_1))Q(m_2)Q(m_3)Q(m_4)$, $Q(PS_6) = Q(ue\ he_1 \overline{he}_2 he_3) = Q(ue)Q(m_1)(1 - Q(m_2))Q(m_3)$ and $Q(PS_9) = Q(ue\ he_1 he_2 he_3 he_4) = Q(ue)Q(m_1)Q(m_2)Q(m_3)Q(m_4)$. The frequency associated to the undesired event was $Q(ue) = 1.1 \times 10^{-7}$. **Table 6** gives the best result obtained by *FSA-DE* method for *ETO* case study.

Table 6 - *ETO* case study, *FSA-DE* results

m_1	m_1	m_1	m_1	C_{tot}
2	1	3	2	2810.0

CONCLUSIONS

The paper investigated the suitability of the *FSA-DE* optimization method for solving reliability optimization problems. *FSA-DE* is advantageous over other optimization methods since it is an almost parameter free method. First a known *RAP* case study was investigated by applying the *FSA-DE* optimization method and the obtained results were compared with other results published in the literature.

Finally, for illustrative purposes, two new optimization case studies were built inspired from published information: a *FTO* case study and an *ETO* case study, and the numerical optimization results obtained by applying the *FSA-DE* method were presented. The study proved that *FSA-DE* is a competitive optimization method for solving reliability optimization problems.

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