OPTIMIZATION OF GRAVITY TURN TO PRESCRIBED CIRCULAR ORBIT BY MODULATION OF THRUST FOR SSTO, USING DIFFERENTIAL EVOLUTION

Mihai-Victor PRICOP[*,**], **Ionuț BUNESCU**[*,**],**Mihai-Vlăduț HOTHAZIE**[*,**],
Mihăiță Gilbert STOICAN[*], **Irina Carmen ANDREI**[*]

[*]National Institute for Aerospace Research "Elie Carafoli", Bucharest, Romania (pricop.victor@incas.ro)
[**]Politehnica University of Bucharest, Faculty of Aerospace Engineering, Romania

DOI: 10.19062/2247-3173.2024.25.22

Abstract: *Single Stage to Orbit vehicles is present in literature for decades. Proposals from various companies emerged, but never happened. The feasibility of a proposed design to achieve the orbit in one stage can easily be approached with a numerical procedure that relies on a robust optimizer. A numerical demonstrator is proposed, based on a trajectory propagator and Differential Evolution (DE) optimizer. The optimization problem consists in achieving circular orbit parameters: altitude, speed and flight path angle at the end of the burn, by modulating thrust that is a polyline and the initial flight path angle. Norm 2 of difference between effective parameters and target ones is minimized. Although the DE optimizer handles constraints according the superiority of feasibility method, there is no benefit in using this approach. After many attempts, the problem has been formulated by adding weighted squares of constraints to the optimization function which is more or less equivalent to a penalty approach. The study enabled robust formulation, a certain parameterization of thrust versus time and the size of population to achieve good convergence.*

Keywords: *space launcher, gravity turn, differential evolution, optimization, circular orbit*

1. PROBLEM DESCRIPTION

A 2D model is used to generate an optimal reference trajectory than can be further used for vehicle development and for checking of feasibility of various proposed vehicles. The dynamic model does not consider lift force and the wind due to planet rotation [1].The main objective is to get an optimal trajectory, using a stochastic optimizer, that is Differential Evolution [3] implemented by Prof. Feng-Sheng Wang, available at [5]. This code has been modified to embed constraints handling according to Superiority of Feasibility method [6]. The dynamic system includes the mass equation of the vehicle, that brings the fuel mass flow rate Eq. (1).

$$
\begin{aligned}
\dot{v} &= -g sin(\gamma) - \frac{D}{m} + \frac{T}{m} \\
\dot{\gamma} &= \frac{g}{v} cos(\gamma) + \frac{v}{r} cos(\gamma) \\
\dot{v} &= \frac{v}{r} cos(\gamma) \\
\dot{r} &= v\, sin(\gamma) \\
\dot{m} &= -\frac{T}{I_{sp}}
\end{aligned}
\tag{1}
$$

The dynamic system is initialized with the parameters described in Table 1.

Table 1. Initial conditions

Parameter	**Description**	**Unit**	**Remark**
$v(t=0)$	Initial velocity	[m/s]	
$\gamma(t=0)$	Flight path angle	[deg]	Optimization variable
$\nu(t=0)$	Polar coordinates: angle	[deg]	
$r(t=0)$	Polar coordinates: radius	[m]	
$m(t=0)$	Initial mass	[Kg]	

The optimization variables are: flight path angle γ_0, fuel mass m_{fuel}, and the thrust vector T_i, that is a set of values linearly interpolated in time, where the timestep in between two successive values is a parameter.

After many attempts, the optimization function was set to embed the constraints as weighted penalties. A vector of 6 components is defining an error/performance function such that the final objective function is the norm 2 of the given vector. Penalties have been applied according to the superiority of feasibility method, but this approach failed. On the other side the penalty method went smooth and manual setting of weights values proved to work after a number of attempts, even with a not normalized weight vector.

$$f_1 = \frac{v_f - v_T}{v_T} \tag{2}$$

$$f_2 = \frac{r_f - r_E - (r_T - r_E)}{r_T} \tag{3}$$

$$f_3 = \gamma \tag{4}$$

$$f_4 = \frac{\left(\frac{dv}{dt}\right)_{max} - 5}{5} \tag{5}$$

$$f_5 = \frac{m_{pl} + x_2\left(1 + f_{empty}\right)}{80 m_{pl}} \tag{6}$$

$$f_6 = \frac{1}{100(T_{max} - T_{min})} \sum_{i=1}^{n_T+1} |T_{i+1} - T_i| \tag{7}$$

Optimization vector can be written as $x^T = \left[\gamma_0, m_{fuel}, T_i\right]^T$. The length of vector T is variable, because the burn time is implicit: thrust is set to zero when the fuel is completely spent, considering Eq. (1) for mass. It is this time value when the flight parameters have to zero the constraints presented in Eq. (2) – Eq. (7), that are further explained in Table 2. For simplicity the orbit is assumed to be circular, of imposed altitude $h_t = r_f - r_E$, where r_f is the polar radius of trajectory at the end of burn, while r_E is the radius of Earth. For circular orbit, the required speed $v_T$is immediately calculated from Eq. (8). Reaching the proper speed by the optimizer is quite sensitive and requires the largest weight, as in Table 2.

$$v_T = \sqrt{\frac{1.9929 \cdot 10^{14}}{r_E + h_T}} \tag{8}$$

Any launcher during the ascent must keep the maximum acceleration bounded, in order to protect itself and the payload, be it material or human. The bound for maximum longitudinal acceleration is set to $5g$, as described in Eq. (5). This constrained is quite easy to be achieved and has a small weight, as in Table 2.

Minimization of mass fuel is connected to the mass of the entire vehicle as in Eq. (6), where x_2, fuel mass, payload mass m_{pl} and empty mass ratio $f_{empty} = 0.07$ are used to calculate the takeoff mass, that is normalized to $80m_{pl}$.

Preliminary results showed quite noisy thrust time profiles. One way to improve this is by increasing the time intervals on which thrust is changed to a value like 20s. Another way is more complex, by minimizing the Total Variation of thrust, as in Eq. (7), that is also normalized to a reference quantity.

Table 2. Optimization function component and their weights

No.	Objective function components	Weight	Remark
1	Eq. (2)	3.0	Orbital velocity relative error
2	Eq. (3)	2.0	Altitude relative error
3	Eq. (4)	1.5	Flight path angle
4	Eq. (5)	0.1	Limitation of maximum acceleration
5	Eq. (6)	0.2	Total mass at take-off
6	Eq. (7)	2.2	Total Variation of Thrust, scaled

$$Obj = \sqrt{\sum_{i=1}^{6} f_i^2} \tag{9}$$

The gravitational acceleration is computed according to the standard power law in this 2D model. The atmospheric parameters are extracted from the US 76 standard [2], with altitude increments of 50m. A look-up table of 20000 lines is stored in memory and linear interpolation is performed in between two successive definition altitudes.

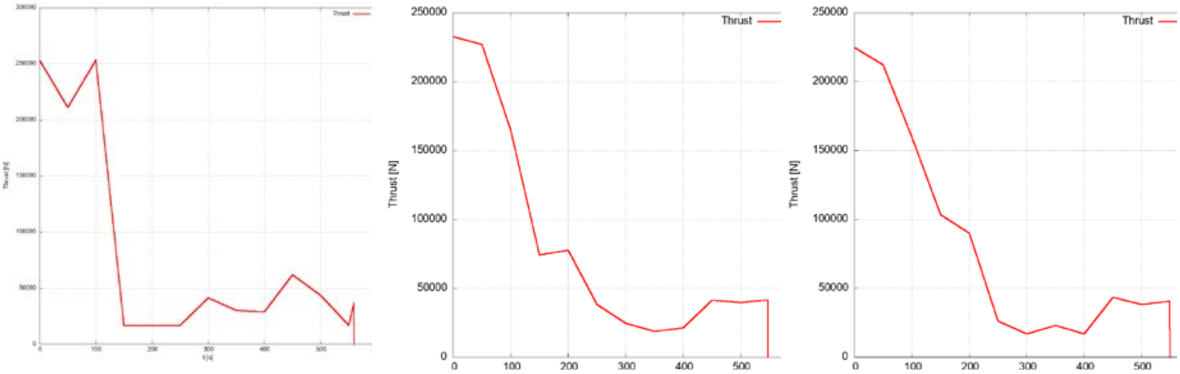
Finally, the objective function is introduced in Eq. (9), as the norm of objective and constraint terms described in Eq. (2)-Eq. (7), further detailed in Table 2.

2. NUMERICAL PROCEDURE

A fourth order predictor-corrector Adams Moulton time integration scheme is used, while correction is applied iteratively, maximum three times, for reaching an imposed tolerance of 10^{-10},for a time step of 0.2s, making the scheme implicit. Time integration is started with Runge-Kutta of order 6. The DE optimizer is used, considering 25 optimization variables. Depending on the fuel burn time, some of the variables are simply not used. The population size has to be large, at about 1000 elements to achieve repeatable results. Min and max bounds are considered for all optimization variables. The objective function is essentially an existing propagator, converted to a function, linked to the optimizer. The code is written in Fortran 95 and is compiled with gfortran. Plots are automatically generated by calling gnuplot, for which scripts are previously prepared. For proper work, an initial velocity of the vehicle is imposed, at 10 m/s.Optimization itself takes about 700s on a 2024 Macbook M3 plus.

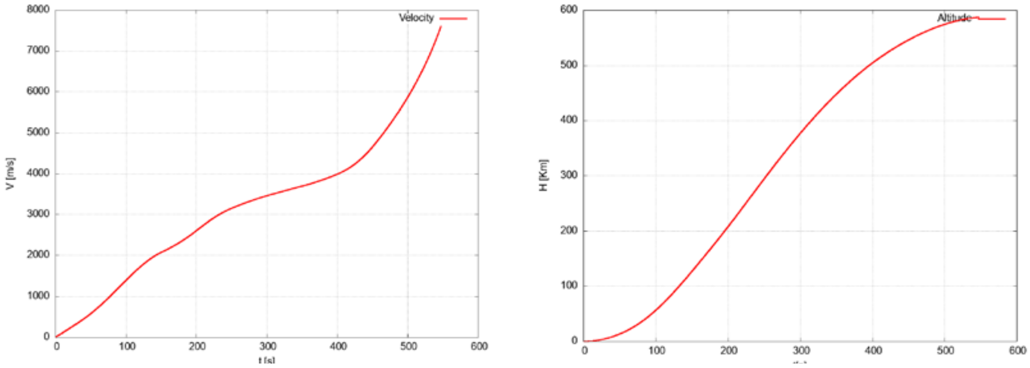
3. RESULTS

Tunning of objective function weights takes some time. The SSTO vehicle in our case is a micro-launcher, for a payload mass of 100 Kg, while an Isp=400s has been found good enough, for the mentioned empty mass fraction. Regardless of the thrust profile in time and other details, the takeoff mass is around 11860 Kg. The orbit is set to 600 Km.



a) b) c)

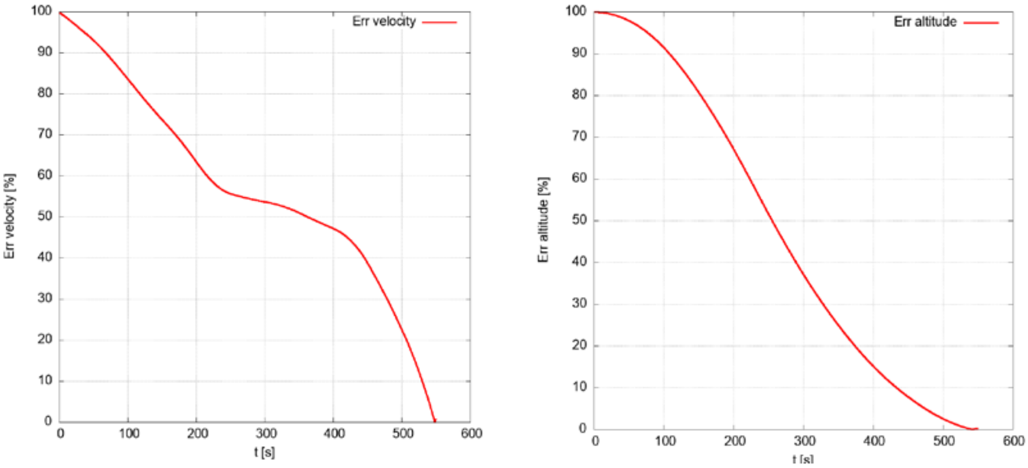
FIG. 1 Thrust profiles a) no treatment, b) and c) TVD penalty in objective function



a) b)

FIG. 2 Velocity and altitude profiles

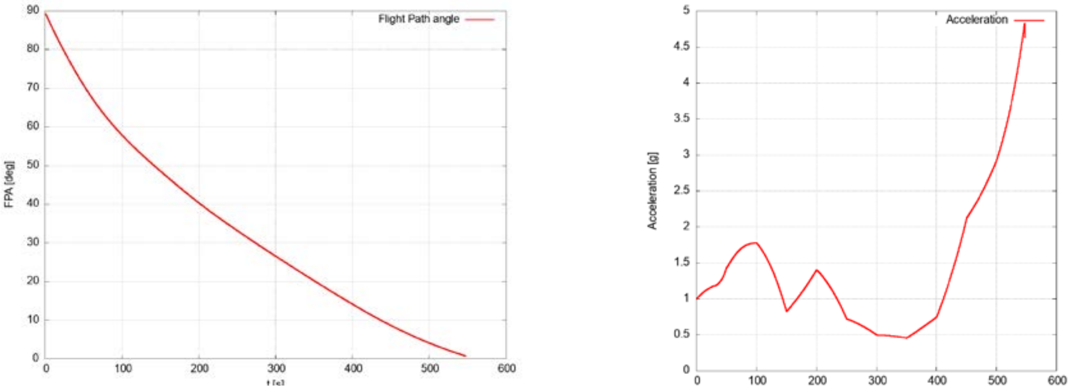
The minimization of thrust TV is producing some improvements, as in Fig. 1, although this can be further improved to a more realistic throttling of a rocket motor. The thrust profiles suggest a coast time, or a two-stage architecture of vehicle.



a) b)

FIG. 3 Relative error in velocity and altitudes versus target values

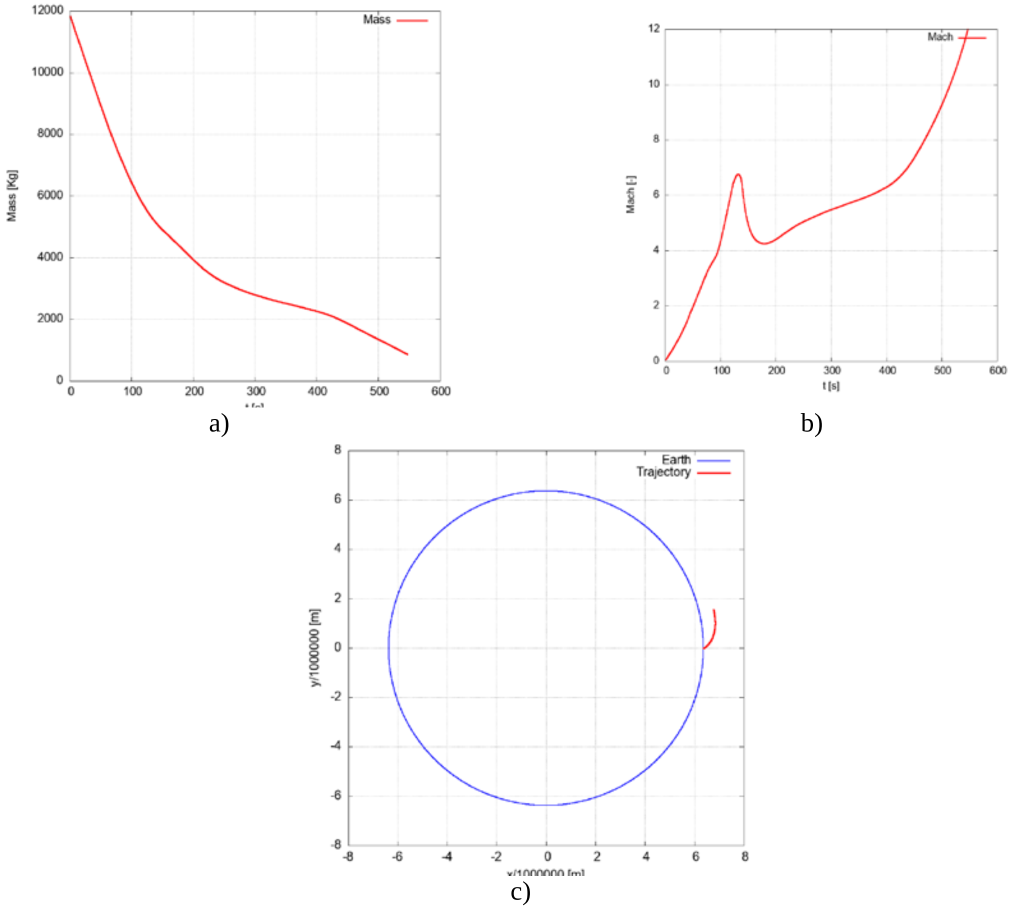
Velocity and altitude profiles are in Fig 2, while their errors versus target values are in Fig. 3. The noisy thrust profile even after the TV treatment is visible in the velocity plots Fig.2 a) and Fig. 3 a).



a) b)

FIG. 4 Flight path angle a), acceleration kept under control b)

The flight path angle is close to a zero value at the end of the burn, Fig. 4 a). A further trajectory propagation would show that the quality of orbit is not good enough, as in reality this must be achieved/maintainedby the useof the flight controller. For further improvement, the objective function may embed a quality index build as error for a couple of propagated orbits, to enforce a better accuracy, but this means a significant increase in computation time, that may be alleviated by a simplified model: no atmosphere function calling. Maximum acceleration is kept under 5g as in Fig. 4 b).



a) b)

c)

FIG. 5 Evolution of mass a), Mach number according to US 76 standard atmosphere b), trajectory c)

4. CONCLUSIONS

The paper presented the problem formulation and gave solution procedure details for a notional trajectory of an SSTO vehicle, for a payload of 100 Kg, target orbit of 600 Km, for an Isp of 400s.At national level there are some good results in this field, although for a multi-stage vehicle, in papers like [7] and [8], but the problem of SSTO was not addressed considering perturbations, according to our knowledge. The penalty formulation of objective functions made sense with the DE optimizer, for a rather large size of population. Repeated runs show dispersion of results in time histories, but the global performance is quite the same, especially in the fuel and respectively total mass of vehicle for the given mission. The thrust profile shaking was damped with a Total Variation constraint, implemented as a penalty in the objective function, this concept being inspired from the CFD numerical methods. A number of runs is needed to setup Isp, fuel mass, empty vehicle and payload masses. Spline interpolation of thrust profile is considered for the future, or more realistic, a differential model, mimicking the propulsion system behavior.The initial flight path angle is an optimization variable, but this may be removed against an imposed value. Other optimizers are considered, like MIDACO, plus conventional methods for solution refinement.

5. ACKNOLEDGEMENT

This work was fully supported by the contract Nucleu PN-23-17-07-02 funded by the Romanian Ministry of Research, Innovation and Digitalization.

REFERENCES

[1] A. Pechev, Reentry Dynamics. Lecture, University of Surrey, 2014, UK;

[2] U.S. STANDARD ATMOSPHERE, 1976, NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION, NATIONAL AERONAUTICS AND SPACE ADMINISTRATION, UNITED STATES AIR FORCE, Washington D.C., October 1976;

[3] Storn, Rainer & Price, Kenneth. (1997), *Differential Evolution - A Simple and Efficient Heuristic for Global Optimization over Continuous Spaces*, Journal of Global Optimization. 11. 341-359. 10.1023/A:1008202821328;

[4] M. Ursu, N. Ursu-Fischer, *Metode numerice în tehnică*, 2019, ISBN: 978-606-17-1450-6, Casa Cărții de știință;

[5] http://mirror.krakadikt.com/2004-11-13-genetic-algorithms/www.icsi.berkeley.edu/%257Estorn/ code.html;

[6] Z. Kajee-Bagdadi, *Differential Evolution Algorithms for Constrained Global Optimization*, A thesis submitted to the Faculty of Science, University of the Witwatersrand, Johannesburg in fulfillment of the requirements for the degree of Master of Science, Johannesburg, 2007 https://wiredspace.wits.ac.za/server/api/core/bitstreams/dcb1417f-9a28-450a-af19-7f3db82505f8/content;

[7] A.I. Onel, T.V. Chelaru, *Aerodynamic assessment of axisymmetric launchers in thecontext of multidisciplinary optimisation*, INCAS BULLETIN, Volume 12, Issue 1/ 2020, pp. 135 – 144, (P) ISSN 2066-8201, (E) ISSN 2247-4528, DOI: 10.13111/2066-8201.2020.12.1.13;

[8] A.I. Onel, T.V. Chelaru, *Trajectory assessment and optimisation in the context ofsmall launcher design*, INCAS BULLETIN, Volume 12, Issue 2/ 2020, pp. 117 – 132, (P) ISSN 2066-8201, (E) ISSN 2247-4528, DOI: 10.13111/2066-8201.2020.12.2.10.