ACCURACY OF NAVIGATION COMPLEXES

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Abstract: *Airplane flight separation is defined for aircraft equipped with complex navigation systems built on the basis of complexation of navigation information derived from inertial navigation system, doppler speed meter and air signal system.*

Keywords: *RNAV, RNP, accuracy of navigation complexes, separation.*

1. INTRODUCTION

In the context of the Single European sky, it is necessary to define the capabilities of navigational complexes of RNPs (required navigation performances) that allow the flight to be performed under RNAV - Area Navigation (a method that allows navigation of an aircraft flight route within or special navigation equipment (terrestrial or satellite) serving as points of reference, or within the capabilities of autonomous airborne devices or a combination of both methods).

The problem with the deployment of zone navigation consists not only in providing a flight on any route but in that the accuracy of the route maintenance corresponds to the requirements established for the area. These requirements are called Required Navigation Performance (RNP). As a result, zonal navigation issues are closely intertwined with RNP problems. RNP is now considered as a tool for technical and law regulation of RNAV flight operations.

Depending on the requirements for the accuracy of maintaining the set route and the functional requirements for the on-board equipment, the following RNAV names are used:

- B-RNAV (Basic RNAV) RNP5 - Basic Area Navigation;
- P-RNAV (Precision RNAV) RNP-1 - accurate Area navigation;
- RNP - RNAV - Area navigation with necessary navigation features.

2. ACCURACY REQUIREMENTS FOR NAVIGATION COMPONENTS FLEETED WITH BASIC AREA NAVIGATION (B-RNAV) CONDITIONS

RNAV certified airplane is navigation equipment ensuring that the pre-determined flight line maintenance is equal to or better than 5 NM for 95% of flight time (RNP 5). This value includes the source signal error, the onboard receiver error, the system display error, and the technical flight error. The system shall ensure that the probability of loss of all navigational information or the presentation of misleading navigational information is less than $1x10^{-5}$ per hour of flight.

The minimum level of sufficiency and integrity required for RNAV systems for their use in the relevant airspace may be provided by a single on-board system including one or more sensors, a RNAV computer, a control and display unit and a navigation display, provided that system is continuously controlled by the crew and in the event of a system failure, the aircraft maintains its ability to navigate over terrestrial navigation facilities such as VOR, DME and NDB.

When performing operations on the same road line, the minimum RNAV longitudinal separation is 150 km (80 NM).

For airplanes in a horizontal flight, set or downhill on roadway lines, a minimum separation distance of 93 km (50 NM) is used with the required navigational characteristics RNP 10.

3. CRITERIA FOR THE ACCURACY OF THE NAVIGATION COMPLEX IN LONGITUDINALLY ELEVATION

One of the main tasks solved by the navigation complexes is to maintain a set distance or a temporary interval between planes flying one route at the same altitude.

In the first way, it is necessary to pass through a specified route point, at a set time, with a fault of no more than ± b.

In the second, the passage through a certain route point located at a distance S from the exit point must be time-lag error ±τ.

The accuracy in longitudinal direction when defining a boundary by distance is expressed by the formula:

$$\Phi_s(t) = \Phi^* \left[\frac{b}{\sigma_s(t)}\right] = \sqrt{\frac{2}{\pi}} \int_0^{\frac{b}{\sigma_s(t)}} e^{-\frac{x^2}{2}} \partial x; \tag{1}$$

Longitudinal accuracy when setting a boundary over time is expressed by the formula:

$$\Phi_\tau(t) = \Phi^* \left[\frac{b}{\sigma_\tau(t)}\right] = \sqrt{\frac{2}{\pi}} \int_0^{\frac{\tau}{\sigma_\tau(t)}} e^{-\frac{x^2}{2}} \partial x. \tag{2}$$

where: b - a long distance corridor is set; t - given longitudinal corridor at time; Ds (t) = σ_s^2 (t) - the dispersion of the error of determining the longitudinal coordinate S at time t; Dt (t) = σ_t^2 (t) - dispersion of the time t of the passage of the airplane through point S.

4. CRITERIA FOR THE ACCURACY OF NAVIGATION COMPLEXS IN SIDE ELEVATION

In a side elevation, the navigation task consists of providing fields on the specified route. The quality of solving the task is characterized by the probability that, at a given stretch of the flight path, the aircraft will never leave the boundary of the corridor. This probability can be determined by considering a sufficient number of N aircrafts and assuming that they fly at the same speed on the same route and are equipped with one and the same navigation complexes. All airplanes at the beginning of the route section considered (t =0) are corrected (Fig. 1).

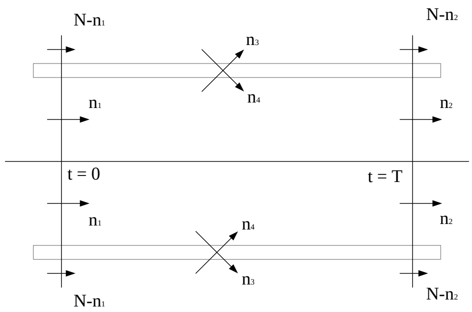

FIG. 1 Schematic of the possible transitions of airplanes at the boundary of the air routes over time (0, T)

Of the total number of airplanes N after the correction, only n_1 planes are found in the corridor, and (n-n_1) are outside. In the process of executing the flight time t. in the range [0, T], from planes in the corridor (n_1) n_3 aircraft will leave the corridor. At the same time, part of the aircraft outside the corridor (n4) will enter it. At the end of the time interval T for the flight of the examined section in the corridor, n_2 of the number of planes and n (n-n_2) of the number of planes are found. Then $n_1 + n_4 = n_3 + n_2$.

The accuracy of navigational flow for aircraft N can be characterized by the number of planes that have never left the boundaries of the set corridor for time t = (0-T). The relative number of these airplanes will be equal to:

$$\frac{n_1}{N} = \left(1 - \frac{n_3}{n_1}\right), \quad (3)$$

and the probability of the aircraft remaining in the set corridor can be used as a criterion for the accuracy of the navigation complex:

$$P = P_1 - P_3, \quad (4)$$

where: P1 is the probability of aircraft entering the prescribed corridor after the correction is made, and P3 is the probability of aircraft departing beyond the boundary of the corridor during the flight for the set interval of time T.

These probabilities can be expressed by the number of aircraft in the groups considered above:

$$P_1 = \frac{n_1}{N} \quad \text{и} \quad P_3 = \frac{n_3}{N}. \quad (5)$$

For the determination of probability P_3, the theory of rebounds of a random process over a certain level is used. Over time the flight changes some navigation parameters (V, W, UO, H, etc.) under the influence of occasional external and internal factors, resulting in deviation of the aircraft from the set route. The navigation process is random and represents a continuous function of time, therefore it is a differentiable process. The main features of this process are:

- the number of crossings at a fixed level from the bottom up, when the derivative of the function is positive;
- the time when the intersection of the fixed level first occurs;
- the length of the time interval in which the function has a value above the fixed level (when the airplane is outside the air corridor);
- the length of the time interval between the individual jumps of the function;
- the value of the maximum exceedance of the fixed function level (linear side deviation LSD $_{max}$).

These features allow you to evaluate the accuracy and reliability of navigational tasks. For air navigation, it is particularly important to determine the probability that for a certain period of time there will be no rebound of the random process.

This probability is unambiguously related to the P_3 probability of the aircraft leaving the boundaries of a given air corridor. From the point of view of random cascade theory, the probability P_3 represents the average number of bounces of the random function of the T time interval.

The formulated criterion meets the basic requirements to the performance criteria of most systems, the most important of which is the ability to obtain a quantitative efficacy assessment.

The accuracy criterion also satisfies other requirements to the performance criteria.

It is inherent in it: efficiency in a statistical sense, low dispersion and hence high accuracy, completeness of the assessment of punctual efficiency, simplicity and presence of physical meaning.

In order to derive equivalences for punctual efficiency, it is necessary to consider a sufficiently large flow of airplanes along the same road line at the same speed and equipped with the same navigation complexes. For this purpose, it is necessary to consider two sections A (t_1) and B (t_2) of the set section of the road line. (Fig. 2).

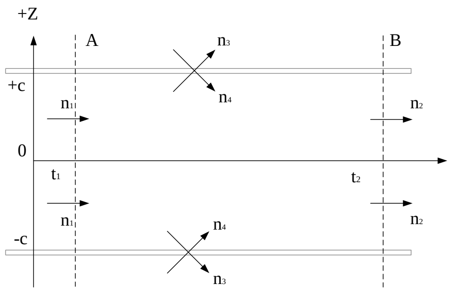

FIG. 2 Schematic of the possible transitions of airplanes at the boundary of the air routes for time $\Delta t = t_2 - t_1$

At point O, all airplanes have completed their correction, and their accuracy in determining aircraft coordinates can be described at the time of intersection of section A (t_1) with the two-dimensional probability distribution density ω (Z, $\dot{Z}$ / t), and section B (t_2) with the probability probability density ω (Z, $\dot{Z}$ / t + Δt) where Z and $\dot{Z}$ are the linear lateral deviation (LOD) and the rate of change of the LSD, and $\Delta t = t_2 - t_1$ is the time difference between the intersection times of the two sections A and B.

Probabilities P and P3 can be determined by one-dimensional and two-dimensional probability densities ω (Z / t) and ω (Z, $\dot{Z}$ / t):

$$P_3 = P_3^+ + P_3^- \tag{6}$$

where:

$$P_3^+ = \int_0^T \omega\left(\frac{c}{t}\right) dt = \int_0^T dt \int_0^{\infty} \omega\left(c, \frac{\dot{Z}}{t}\right) \dot{Z} d\dot{Z},$$

$$P_3^- = \int_0^T \omega\left(\frac{-c}{t}\right) dt = -\int_0^T dt \int_{-\infty}^{0} \omega\left(-c, \frac{\dot{Z}}{t}\right) \dot{Z} d\dot{Z}.$$

P_3^+ и P_3^- are the probabilities of crossing, respectively, the boundaries of the corridor (-c, + c). these probabilities are the same and therefore $P_3 = 2P_3^+$.

The navigation process is a normal random process and the probability densities used are the ratios:

$$\omega\left(\frac{Z}{t}\right)=\frac{1}{\sigma_z\sqrt{2\pi}}e^{-\frac{(Z-m_z)^2}{2\sigma_z^2}}; \qquad (7)$$

$$\omega\left(Z,\frac{\dot{Z}}{t}\right)=\frac{1}{2\pi\sigma_z\sigma_{\dot{z}}\sqrt{1-r^2}}\times\exp\left\{-\frac{1}{2\sqrt{1-r^2}}\left[\frac{(Z-m_z)^2}{\sigma_z}-2r\frac{(Z-m_z)(\dot{Z}-m_{\dot{z}})}{\sigma_z\sigma_{\dot{z}}}+\frac{(\dot{Z}-m_{\dot{z}})}{\sigma_{\dot{z}}^2}\right]\right\} \qquad (8)$$

where: $m_{\dot{z}}=m_{\dot{z}}(t)=\frac{dm_z(t)}{dt}$;

$\sigma_z^2=\sigma_z^2(t)=k_z(t_1t_2)|t_1=t_2=t$;

$\sigma_{\dot{z}}^2=\sigma_{\dot{z}}^2(t)=k_{\dot{z}}(t_1t_2)|t_1=t_2=t=\frac{d^2k_z(t_1,t_2)}{dt_1dt_2}|t_1=t_2=t$;

$r=r(t)=\frac{k_{z\dot{z}}(t_1,t_2)|t_1=t_2=t}{\sigma_z\sigma_{\dot{z}}}=\frac{1}{\sigma_z\sigma_{\dot{z}}}\frac{dk_z(t_1,t_2)}{dt_2}||t_1=t_2=t$;

σ_z and m_z are the mean quadratic deviation and the mathematical expectation of the rate of change of the linear lateral deviation (LSD) at the moment (t);

$k_{\dot{z}}(t_1t_2)$ is the correlation function of LSD errors;

$k_{z\dot{z}}(t_1t_2)$ is the correlation function of Z and $\dot{Z}$;

r is the intercorrelation function of the Z and $\dot{Z}$ at the moment (t).

By replacing the probability density ω (Z / t) from (5) is obtained:

$$P_1=\frac{1}{\sqrt{\pi}}\int_0^{\frac{c}{\sigma_z\sqrt{2}}}e^{-q^2}dq-\frac{1}{\sqrt{\pi}}\int_0^{\frac{c}{\sigma_z\sqrt{2}}}e^{-q^2}dq,$$

Where $q=\frac{Z-m_z}{\sigma_z\sqrt{2}}$ is the new variable.

$$P_1=\frac{1}{2}erf\left(\frac{c}{\sigma_z\sqrt{2}}\right)-\frac{1}{2}erf\left(\frac{c}{\sigma_z\sqrt{2}}\right)=erf\left(\frac{c}{\sigma_z\sqrt{2}}\right) \qquad (9)$$

To find the probability P_3^+ it is necessary to replace the probability density of (6) and integrate:

$$\omega\left(\frac{c}{t}\right)=\int_0^\infty\omega\left(c,\frac{\dot{Z}}{t}\right)\dot{Z}d\dot{Z}$$

$$I_1=\int_0^\infty\frac{1}{2\pi\sigma_z\sigma_{\dot{z}}\sqrt{1-r^2}}exp\left\{-\frac{1}{2(1-r^2)}\left[\frac{(c-m_z)^2}{\sigma_z^2}-\frac{2r(c-m_z)(\dot{Z}-m_{\dot{z}})}{\sigma_z\sigma_{\dot{z}}}+\frac{(\dot{Z}-m_{\dot{z}})}{\sigma_{\dot{z}}^2}\right]\right\}\dot{Z}dZ$$

$$\begin{aligned}I_1&=0+\frac{A}{2p}exp\left[-p\left(\frac{m_{\dot{z}}}{\sigma_{\dot{z}}}+q\right)^2\right]+\frac{A}{\sqrt{p}}\left(\frac{m_{\dot{z}}}{\sigma_{\dot{z}}}+q\right)\times\left[\frac{A}{\sqrt{\pi}}\int_0^\infty exp(-v^2)\,dv+\int_{-\sqrt{p}\left(\frac{m_{\dot{z}}}{\sigma_{\dot{z}}}+q\right)}^\infty exp(-v^2)dv\right]= \qquad (10)\\&=\frac{A}{2p}exp\left[-p\left(\frac{m_{\dot{z}}}{\sigma_{\dot{z}}}+q\right)^2\right]+\frac{A\sqrt{\pi}}{2\sqrt{p}}\left(\frac{m_{\dot{z}}}{\sigma_{\dot{z}}}+q\right)\times\left\{1-erf\left[-\sqrt{p}\left(\frac{m_{\dot{z}}}{\sigma_{\dot{z}}}+q\right)\right]\right\}=\\&=\frac{A}{2p}exp\left[-p\left(\frac{m_{\dot{z}}}{\sigma_{\dot{z}}}+q\right)^2\right]+\frac{A\sqrt{\pi}}{2\sqrt{p}}\left(\frac{m_{\dot{z}}}{\sigma_{\dot{z}}}+q\right)\times\left\{1+erf\left[\sqrt{p}\left(\frac{m_{\dot{z}}}{\sigma_{\dot{z}}}+q\right)\right]\right\}\end{aligned}$$

After replacing the expressions for p, q and A, there is obtained:

$$\omega\left(\frac{c}{t}\right)=\frac{1}{2\pi}\frac{\sigma_{\dot{z}}}{\sigma_z}e^{-\frac{(c-m_z)^2}{2\sigma_z^2}}\left[\sqrt{1-r^2}exp\left[-\frac{1}{2\sqrt{1-r^2}}\left(\frac{\sigma_{\dot{z}}}{\sigma_z}+\frac{r(c-m_z)}{\sigma_z}\right)^2\right]\right.$$

$$\left.+\sqrt{\frac{\pi}{2}}\left(\frac{\sigma_{\dot{z}}}{\sigma_z}+\frac{r(c-m_z)}{\sigma_z}\right)\left[1+erf\left[\frac{1}{2\sqrt{1-r^2}}\left(\frac{\sigma_{\dot{z}}}{\sigma_z}+\frac{r(c-m_z)}{\sigma_z}\right)\right]\right]\right]$$

$$P_3=2P_3^+=2\int_0^T\omega\left(\frac{c}{t}\right)dt$$

5 RESULTS

The test shall be carried out for the following conditions: W=600 – 900 km/h; S=300 – 2000km. c = 10NM.

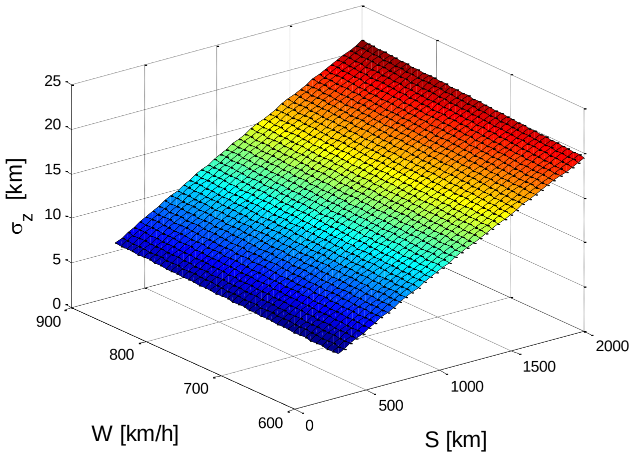

FIG. 3 Dependence of the side deviation σ_z from the speed W and the path S

Table 1

σ_z, km	**S=300, km**	**500**	**1000**	**1500**	**2000**
W=600, km/h	4,903	6,800	11,284	15,486	19,505
700	5,236	7,151	11,709	15,998	20,097
800	5,568	7,495	12,112	16,475	20,648
900	5,898	7,835	12,499	16,926	21,165

As the speeds W and S increase, the mean square radial error σ_z increases, varying from 4,903 km to 21,165 km (Tab 1, Fig 3).

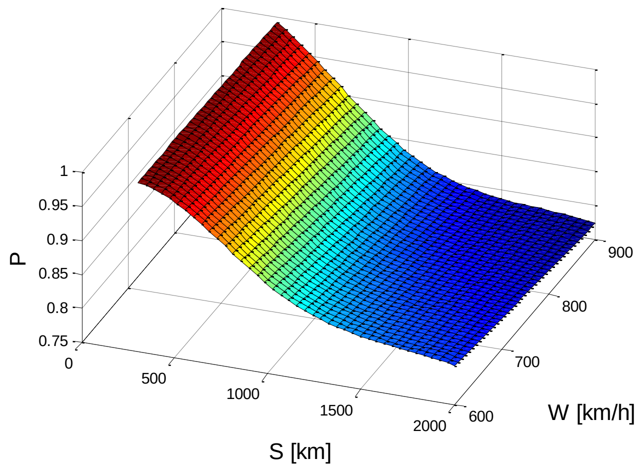

FIG. 4 Probability dependence P for remaining in a corridor ± 5NM from W and S

Table 2

P	**S=300, km**	**500**	**1000**	**1500**	**2000**
W=600, km/h	0,9989	0,9792	0,8793	0,8268	0,8071
700	0,9976	0,9700	0,8625	0,8122	0,7944
800	0,9952	0,9595	0,8467	0,7990	0,7833
900	0,9917	0,9479	0,8318	0,7870	0,7733

With the increase of speed W and S, the probability of the aircraft remaining in a corridor with a width of ± 5 NM decreased and varied within the range of 0.9989 to 0.77733 km (Tab 2, Fig 4).

Table 3

σ_s, km	**S=300, km**	**500**	**1000**	**1500**	**2000**
W=600, km/h	12,007	20,005	40,004	60,005	80,005
700	12,007	20,005	40,004	60,004	80,004
800	12,007	20,004	40,003	60,003	80,004
900	12,007	20,004	40,003	60,003	80,003

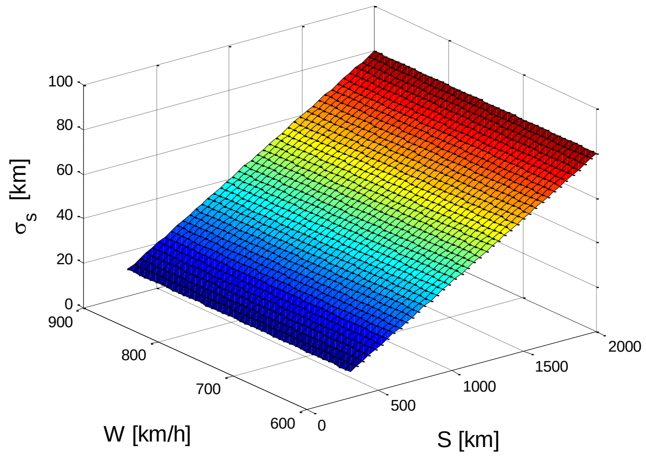

FIG. 4 Dependence of longitudinal deviation σs from speed W and path S

Table 4

$P_{(80)}$	**S=300, km**	**500**	**1000**	**1500**	**2000**
W=600, km/h	1,000	1,000	1,000	0,988	0,939
700	1,000	1,000	1,000	0,988	0,939
800	1,000	1,000	1,000	0,988	0,939
900	1,000	1,000	1,000	0,988	0,939

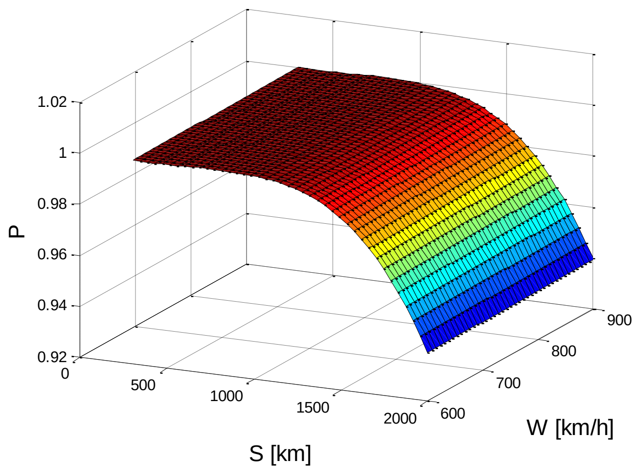

FIG. 5 *Probability dependence P to remain in a corridor ± 80NM from W and S*

Table 5

$P_{(50)}$	S=300, km	500	1000	1500	2000
W=600, km/h	1,000	1,000	0,981	0,883	0,760
700	1,000	1,000	0,981	0,883	0,760
800	1,000	1,000	0,981	0,883	0,760
900	1,000	1,000	0,981	0,883	0,760

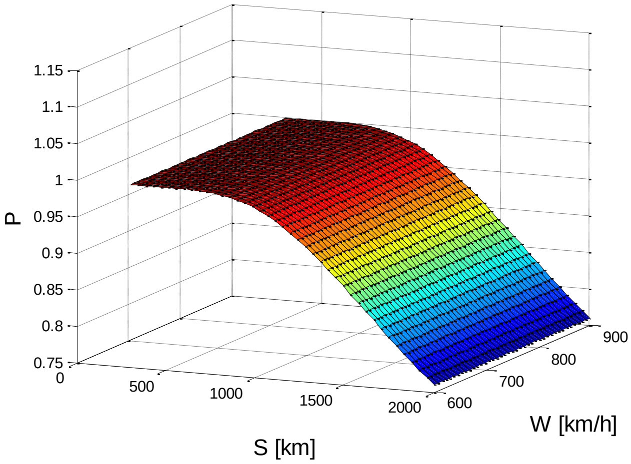

FIG. 6 Probability dependence P to remain in a corridor ± 50NM from W and S

5. CONCLUSIONS

On the basis of the results obtained for aircraft equipped with navigation systems built on the basis of complexation of navigation information derived from Inertial Navigation System, Doppler Speed Meter and Air Signal System, the following conclusions can be drawn:

- Airplanes meet the requirements of B-RNAV for 1000km. route length;
- Airplanes meet the requirements of RNP10 RNAV for 500km. route length;
- Airplanes meet the requirements of B-RNAV (RNP5) up to 500 km. route length;
- for a route longer than 500 km. B-RNAV (RNP5) requirements are not met;
- the aircraft do not meet the requirements of the P-RNAV (RNP1).

For a RNAV flight it is necessary that the aircraft be equipped with navigation systems that complex navigation information from an inertial navigation system, Doppler Speed Meter, Air Signaling System, and Satellite Navigation System.

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