THE STABILITY OF THE CONTROLLABILITY BY TWO SCALE HOMOGENIZATION

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Abstract: *In this paper we use the HUM method and we performed an exact internal control which operates on the border of the holes from an* ε *- periodic perforated domain. Using the two-scale convergence method, we obtain a homogenized hyperbolic problem controlled this time on the whole homogenized domain (without holes) by a surface integral from the limit of the control.*

Keywords: ε*-periodic perforated domain, two-scale convergences, HUM method, exact internal control, homogenization.*

1. INTRODUCTION

In this article, we study a stability problem: what becomes a hyperbolic problem on an ε- periodic perforated domain which is controlled on the border of the holes? Will the property of controllability by homogenization be kept after $\varepsilon \to 0$ (where ε is the distribution period of the holes in the domain) ?

The initial control is found with the HUM method, but, now, it is applied on a perforated domain.

The answer to the last question is an affirmative one. The novelty of the result consists in the fact that the surface integral from the limit of the control is the exact internal control for the homogenized hyperbolic problem – the limit of the hyperbolic initial problem which has got an exact control. Another difference, from other similar articles dealing with the same topic, is the two-scale method, in contrast with the energetic method of Tartar in [8], which cannot be applied in the case of the perforated domain, from this article, a domain which has the property that the holes intersect the border of the homogeneous domain (without holes).

2. THE GEOMETRY OF THE DOMAIN

Let be Ω an open bounded domain from $\mathbb{R}^n (n \geq 3)$ with smooth border $\partial\Omega$. Let be the representative cell of the following form, $Y = (0, l_1) \times \cdots \times (0, l_n)$, from $\mathbb{R}^n$. We use the following notation from [3] for defining the ε- periodic perforated domain. Let be $T \subset Y$ an open domain so that $\overline{T} \subset Y$. T is called hole.

For any $\varepsilon > 0$ and $k \in \mathbb{Z}^n$ we denote by $T_\varepsilon^k = \varepsilon(kl + T)$ are the holes in $\mathbb{R}^n$, where $kl = (k_1 l_1, ..., k_n l_n)$. We denote by $T_\varepsilon = \bigcup \left\{ T_\varepsilon^k \,\middle|\, T_\varepsilon^k \cap \Omega \neq \varnothing, k \in \mathbb{Z}^n \right\}$ that represent the holes which are in Ω or these intersect the border of Ω. We denote by $\Omega_\varepsilon = \Omega \setminus \overline{T}_\varepsilon$ the ε-

periodic perforated domain. The set $\Sigma_\varepsilon = \bigcup\left\{\partial T_\varepsilon^k \,\middle|\, T_\varepsilon^k \cap \Omega \neq \emptyset, k \in \mathbb{Z}^n\right\}$ represents the border of the holes, the difference from [3] consists in the fact that the holes intersect $\partial\Omega$. We denote by $Y^* = Y \setminus \overline{T}$.

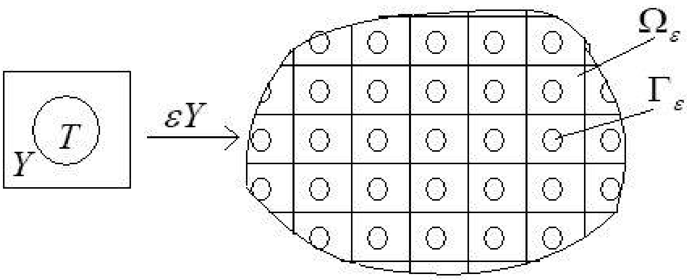

FIG. 1. The structure.

3. THE STATEMENT OF THE PROBLEM

Using the HUM method we obtain the problem that is verified by the exact internal control in the domain $\Omega_\varepsilon \times (0,T)$:

$$
\begin{aligned}
&u_\varepsilon'' - \frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial u_\varepsilon}{\partial x_j}\right) + q_\varepsilon u_\varepsilon = 0, \text{ in } \Omega_\varepsilon \times (0,T) \\
&u_\varepsilon = 0, \text{ on } \partial\Omega \times (0,T) \\
&u_\varepsilon(0) = u_\varepsilon^0,\ u_\varepsilon'(0) = u_\varepsilon^1 \text{ on } \Omega_\varepsilon
\end{aligned}
\tag{1}
$$

where $u_\varepsilon^0 \in U_\varepsilon$, $u_\varepsilon^1 \in L^2(\Omega_\varepsilon)$, $q_\varepsilon \in L^2(\Omega_\varepsilon)$ and U_ε is the Hilbert space defined by $U_\varepsilon = \bigcup\left\{u \in H^1(\Omega_\varepsilon) \,\middle|\, u = 0 \text{ on } \partial\Omega\right\}$ with the associated norm:

$$
\|u\|_{U_\varepsilon} = \left(\sum_{i=1}^{n}\int_{\Omega_\varepsilon}\left(\frac{\partial u}{\partial x_i}\right)^2 dx\right)^{\frac{1}{2}}
\tag{2}
$$

The coefficients a_{ij} are constant and elliptic. With Lax-Milgram theorem the problem (1) has a unique solution. Also, the application $\left(u_\varepsilon^0, u_\varepsilon^1\right) \mapsto \frac{\partial u_\varepsilon}{\partial \nu_\varepsilon}$ is linear and continuous from the space $U_\varepsilon \times L^2(\Omega_\varepsilon)$ to the space $L^2\left(0,T;L^2(\Gamma_\varepsilon)\right)$, so $\frac{\partial u_\varepsilon}{\partial \nu_\varepsilon} \in L^2\left(0,T;L^2(\Gamma_\varepsilon)\right)$. We have to remind ourselves that $\frac{\partial u_\varepsilon}{\partial \nu_\varepsilon}$ represents the derivative of u_ε with respect to the outside normal ν_ε at Γ_ε and it is defined as:

$$
\frac{\partial u_\varepsilon}{\partial \nu_\varepsilon} = a_{ij}\frac{\partial u_\varepsilon}{\partial x_j} n_i
\tag{3}
$$

where $\nu_\varepsilon = (n_1, ..., n_n)$ are the components of the normal.

We denote the exact internal control by $\varphi_\varepsilon = \frac{\partial u_\varepsilon}{\partial \nu_\varepsilon}$ and it controls the following hyperbolic problem:

$$\begin{aligned}
&v_\varepsilon'' - \frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial v_\varepsilon}{\partial x_j}\right) + q_\varepsilon v_\varepsilon = 0, \text{ in } \Omega_\varepsilon \times (0,T) \\
&\frac{\partial v_\varepsilon}{\partial \nu_\varepsilon} = \varepsilon\varphi_\varepsilon, \text{ on } \Gamma_\varepsilon \times (0,T) \\
&\mathrm{v}_\varepsilon = 0, \text{ on } \partial\Omega \times (0,T) \\
&v_\varepsilon(T) = v_\varepsilon'(T) = 0, \text{ in } \Omega_\varepsilon
\end{aligned} \tag{4}$$

With the Lax-Milgram theorem, the problem (4) has a unique solution so that $(v_\varepsilon(0), v_\varepsilon'(0)) \in L^2(\Omega_\varepsilon) \times (U_\varepsilon)'$, where $(U_\varepsilon)'$ is the dual of U_ε. More than this, the application $(u_\varepsilon^0, u_\varepsilon^1) \mapsto (v_\varepsilon(0), v_\varepsilon'(0))$ is linear and continuous from $U_\varepsilon \times L^2(\Omega_\varepsilon)$ to $L^2(\Omega_\varepsilon) \times U_\varepsilon'$.

In next section we homogenize these two problems using the idea from [6] and the two-scale convergence method introduced in [1] and [7].

4. THE HOMOGENIZATION AFTER $\varepsilon \to 0$

The homogenized problems for (1) and (4) are:

$$\begin{aligned}
&(measY^*)u'' - \frac{\partial}{\partial x_i}\left(A_{ij}^{\text{hom}}\frac{\partial u}{\partial x_j}\right) + qu = 0, \text{ in } \Omega \times (0,T) \\
&u = 0, \text{ on } \partial\Omega \times (0,T) \\
&u(0) = \frac{u^0}{measY^*},\ u'(0) = \frac{u^1}{measY^*} \text{ in } \Omega
\end{aligned} \tag{5}$$

where we have the next convergences:

$$\begin{aligned}
&\chi_{\Omega_\varepsilon}(x)u_\varepsilon(x) \xrightarrow{2s} \chi_{Y^*}(y)u(x) \\
&\chi_{\Omega_\varepsilon}(x)q_\varepsilon(x) \xrightarrow{2s} \chi_{Y^*}(y)q(x) \\
&\chi_{\Omega_\varepsilon}(x)u_\varepsilon^0(x) \xrightarrow{2s} \chi_{Y^*}(y)u^0(x) \\
&\chi_{\Omega_\varepsilon}(x)u_\varepsilon^1(x) \xrightarrow{2s} \chi_{Y^*}(y)u^1(x)
\end{aligned} \tag{6}$$

where

$$\chi_{\Omega_\varepsilon}(x) = \begin{cases} 1, x \in \Omega_\varepsilon \\ 0, x \notin \Omega_\varepsilon \end{cases} \tag{7}$$

is the characteristic function of Ω_ε and

$$\chi_{\Omega_\varepsilon}(x) = \chi_{Y^*}\left(\frac{x}{\varepsilon}\right). \tag{8}$$

The homogenized coefficients $A_{ij}^{\hom}$ are given in [2] and [4, 5].

The homogenization of the problem (4). The homogenized controlled problem is:

$$\begin{aligned}
&\left(measY^*\right)v'' - \frac{\partial}{\partial x_i}\left(A_{ij}^{\hom}\frac{\partial v}{\partial x_j}\right) + q(x)v(x) =_\varepsilon v_\varepsilon = 0, \text{ in } \Omega_\varepsilon \times (0,T) \\
&\frac{\partial v_\varepsilon}{\partial \nu_\varepsilon} = \varepsilon\varphi_\varepsilon, \text{ on } \Gamma_\varepsilon \times (0,T) \\
&\mathrm{v}_\varepsilon = 0, \text{ on } \partial\Omega \times (0,T) \\
&v_\varepsilon(T) = v'_\varepsilon(T) = 0, \text{ in } \Omega_\varepsilon
\end{aligned} \tag{9}$$

where

$$\begin{aligned}
&\chi_{\Omega_\varepsilon}(x)v_\varepsilon \xrightarrow{2s} \chi_{Y^*}(y)v(x) \\
&\varepsilon\chi_{\Omega_\varepsilon}(x)\varphi_\varepsilon(x) \xrightarrow{2s} \varphi(x,y)
\end{aligned} \tag{10}$$

and $\varphi(x,y)$ is the limit of the control.

CONCLUSIONS

By comparing the controlled problem by φ_ε and the homogenized problem of it, we can observe: the limit of the control – which operates on the border of the holes – in homogenized problem acts via a surface integral on the whole homogeneous domain (without holes) Ω.

REFERENCES

[1] G. Allaire, *Homogenization and two-scale convergence*, SIAM J, Math. Anal. 23,6, 1482-1518,1992;

[2] D. Cioränescu, J. Saint Jean Paulin, *Homogenization of reticulated structures*, Springer Verlag (New Zork), 1999;

[3] C. Conca, J. I. Diaz, A. Linan, C. Timofte, *Homogeniyation in chemical reactive flows*, Electronic Journal of Differential Equations, No. 40, 1-22, 2004;

[4] C. Gheldiu, *L'homogeneization d'equation des ondes pour structures renforcees*. Buletin Ştiinţific, Universitatea din Piteşti, Nr. 15, 29-38, 2009;

[5] C. Gheldiu, *L'homogeneisation de probleme des ondes pour structures reticulees renforcees vers l'epaisseur du materiau. Controlabilite exacte pour structures reticulees renforcees,* Buletin Ştiinţific – Universitatea din Piteşti, Seria Matematică şi Informatică, Nr. 17 (2011).

[6] V. Komornik, *Exact controllability and stabilization. The multiplies method*, Masson, Paris, 1994;

[7] G. Nguetseng, *Asymptotic analisys for a stiff variational problem in mechanics*, SIAM J, Math. Anal. 21,6, 1394-1414, 1990;

[8] L. Tartar, *Problemes d'homogeneization dans les equations aux derivees partielles*, in Cours Pecot, College de France, 1977;