

## COMPARATIVE ANALYSIS OF TUNING MISSILE AUTOPILOTS USING INTELLIGENT METHODS

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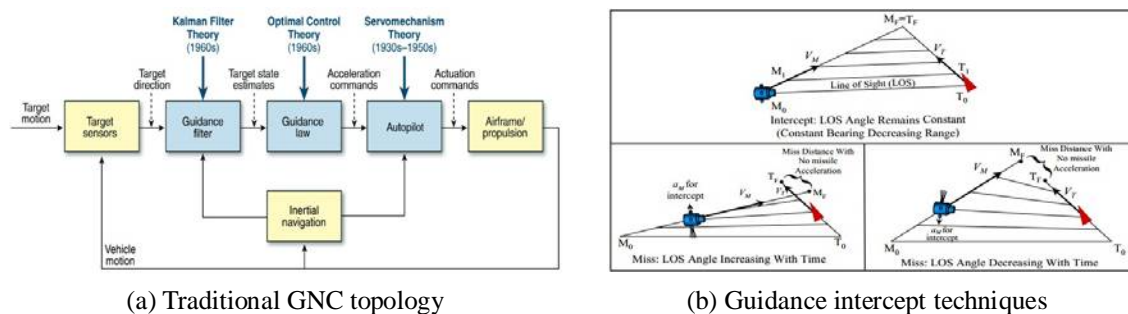
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**Abstract:** This article explores basic technical and design challenges associated with the missile flight control system, including its role in the overall missile system, its subsystems, types of flight control systems, design objectives, and design challenges.

**Keywords:** missile homing loop, feedback control, synthesis techniques for homing guidance.

### 1. INTRODUCTION TO MISSILE AUTOPILOTS

Before we go on to discuss any particular type of guidance system, it is necessary to consider first the overall operation of an entire missile guidance and control system; to divide it into convenient groups of units; and to indicate the general function of each major group so that the operation of the particular units may be understood in relation to the operation of the guidance and control system as a whole [1].

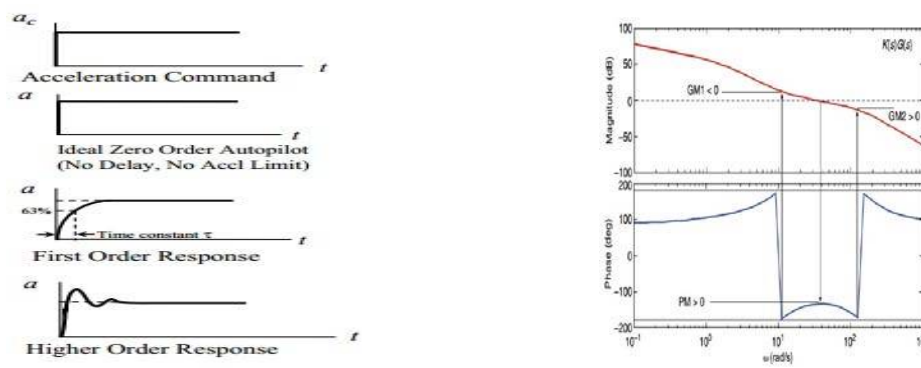


(a) Traditional GNC topology  
**FIG. 1.** The figure show traditional GNC topology and different guidance intercept techniques

As indicated in Fig. 1. (a), the traditional architecture for all fielded guided missile systems are particular examples of the feedback concept. The GNC topology for a guided missile comprises guidance filter, guidance law, autopilot, and inertial navigation components. The inertial navigation system (INS) provides the position, velocity, acceleration, angular orientation, and angular velocity of the vehicle by measuring the inertial linear acceleration and inertial angular velocity applied to the system. The information from the INS is used throughout missile flight to support guidance and flight control functions. The guidance filter receives noisy target measurement data from the homing sensor and estimates the relevant target states. The guidance law takes the instantaneous target-state estimates as input and determines what the interceptor direction of travel should be to intercept the target. It typically is an anticipatory function in that it generates guidance commands to put the missile on a collision course with the target.

The problem is to design a pitch plane autopilot to track the normal acceleration commanded from the guidance system. The autopilot generates fin angle commands which are sent to the tail surface servos. By deflecting the tail fins, they generate aerodynamic forces and moments that maneuver the missile. Rate gyro and accelerometer measurements are processed by the flight control system to close the feedback control loop.

The principal functions of the guidance system are to detect the presence of the target and track it; to determine the desired course to the target; and to produce electrical steering signals which indicate the position of the missile with respect to the required course. Therefore, you can say that the output of the guidance and control system is the actual missile flight path. If there is a difference between the desired flight path (input) and the one the missile is actually on (output), then the control system operates to change the position of the missile in space to reduce the error.



(a) Time response characteristics

(b) Bode plot open-loop frequency response of an acceleration autopilot

**FIG. 2.** Traditional approach for developing GNC system -time and frequency characteristics

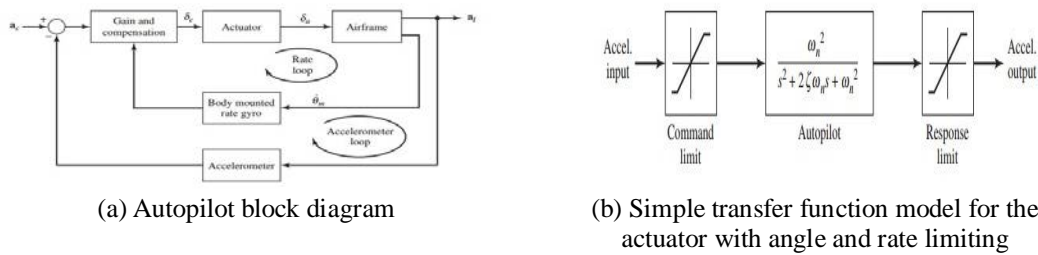
The purpose of an autopilot is to produce lateral missile acceleration  $a$  in response to commanded acceleration  $a_c$  as shown in Fig. 2. An autopilot's time constants the approximate time it takes for the missile to achieve commanded acceleration.

The missile motion in space is completely defined by the acceleration normal to the velocity vector and the rate of change of the velocity magnitude. The commanded normal acceleration is the input to a combination of limiters and transfer functions that simulate the autopilot, control system, and aerodynamics, yielding realized accelerations as the output. Specifically, the commanded acceleration is passed to the autopilot in a body frame sense.

## 2. THE COMMON AUTOPILOT AND MISSILE MODEL

The dynamics of the airframe are governed by fundamental equations of motion, with their specific characteristics determined by the missile aerodynamic response, propulsion, and mass properties. Assuming that missile motion is restricted to the vertical plane (typical for early concept development), the equations of motion that govern the missile dynamics can be developed in straightforward fashion. These equations are sufficient to obtain rough estimates of the impact point. Variations in wind conditions and motor burn as well as heading and attitude control errors would affect actual performance. Adding simple trim aerodynamics with a transfer function representation of the autopilot and a proportional navigation guidance law produces a simulation model.

The gains in the autopilot are scheduled as a function of flight condition to achieve missile stability and command following. The actuator command passes through a second-order transfer function with angle and rate limiters (1).



**FIG. 3.** The autopilot acceleration command from the guidance law and the measured acceleration and body rate as inputs to obtain the actuator command.

A “three-loop” autopilot and simple transfer function model for the actuator with angle and rate limiting used to describe these dynamic as in [5] are:

$$\frac{\delta(s)}{\delta_c(s)} = \frac{\omega_a^2}{s^2 + 2\xi_a\omega_a s + \omega_a^2} \tag{1}$$

The application of the longitudinal (vertical plane) flight control system for a bank to turn missile form a single input multioutput design model. The plant outputs are normal acceleration  $A_z(\text{ft/s}^2)$ , and pitch rate  $q$  (rad/s), and the plant states are  $x = [\alpha \ q \ \delta \ \dot{\delta}]^T$  (angle of attack, pitch rate, fin deflection, and fin rate respectively). The nominal longitudinal airframe dynamics is represented by  $G(s)$ . The deferential equation used to describe these open loop dynamic as in [5] are:

$$\begin{aligned} \dot{\alpha} &= Z_\alpha \alpha + q + Z_\delta \delta_e \\ \dot{q} &= M_\alpha \alpha + M_\delta \delta_e \\ A_z &= VZ_\alpha \alpha + VZ_\delta \delta \end{aligned} \tag{2}$$

Assuming that the actuator is second order system as:

$$\ddot{\delta}_e = -2\xi\omega\dot{\delta}_e - \omega^2(\delta_e - \delta_c) \tag{3}$$

In the state space form, the airframe dynamics are represented by the following state space triple (A, B, C):

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}, \quad A = \begin{bmatrix} Z_\alpha & 1 & Z_\delta & 0 \\ M_\alpha & 0 & M_\delta & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega^2 & -2\xi\omega^2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \omega^2 \end{bmatrix}, \quad C = \begin{bmatrix} VZ_\alpha & 0 & VZ_\delta & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \tag{4}$$

The transfer function matrix is  $G(s) = C(sI - A)^{-1}B$ . The longitudinal missile dynamics form a single input multioutput design model from equation 2-5, the transfer function matrix from the elevon fin deflection command  $\delta_c$  to the normal acceleration  $A_z$  and pitch rate  $q$  is

$$G(s) = \begin{bmatrix} \frac{\omega^2 V(Z_\delta s^2 + Z_\alpha M_\delta - Z_\delta M_\alpha)}{(s^2 - Z_\alpha s - M_\alpha)(s^2 + 2\xi\omega s + \omega^2)} \\ \frac{\omega^2 (M_\delta s^2 + M_\alpha Z_\delta - M_\delta Z_\alpha)}{(s^2 - Z_\alpha s - M_\alpha)(s^2 + 2\xi\omega s + \omega^2)} \end{bmatrix} = \begin{bmatrix} \frac{A_z(s)}{\delta_c(s)} \\ \frac{q(s)}{\delta_c(s)} \end{bmatrix} \tag{5}$$

where  $Z_\alpha, Z_\delta, M_\alpha, M_\delta$  and  $M_q$  are the aerodynamic stability derivatives. The measurements that are available are normal acceleration  $A_z = VZ_\alpha\alpha + VZ_\delta\delta$  (ft/s<sup>2</sup>),  $q$  pitch rate (rad/s). The scalar control input  $u = \delta$  (rad) is the fin angle  $\alpha$  command. Although these differential equations can be solved numerically, an analytical approach often is desirable to fully understand the missile dynamics. Therefore, the equations of motion are linearized around an operating condition so that linear systems theory can be applied.

The above aerodynamics have been linearized and represented a trim  $\alpha$  angle of attack of 16 degrees, Mach number=0.8,  $V=886.78$  (ft/s), an altitude of 4000 (ft.), actuator damping  $\zeta = 0.6$ , and actuator natural frequency  $\omega = 113$  (rad/s). The following parameters are the nominal values of the dimensional aerodynamic stability derivatives;  $Z_\alpha = -1.3046$  (1/s);  $Z_\delta = -0.2142$ (1/s);  $M_\alpha = \pm 47.7109$  (1/s<sup>2</sup>) which were taken from [4]. The sign of  $M_\alpha$  determines the stability of the open loop airframe. When the  $M_\alpha$  is negative the airframe is stable, and when it is positive the airframe is unstable, which occurs when the aerodynamic center of pressure is forward of the center of gravity [5].

### 3. EXPLORE „THREE-LOOP” AUTOPILOT

The three loop pitch/yaw autopilot is used to most guided tactical missiles today as shown in Fig. 4. It has four gains  $K_{DC}, K_A, K_R$  and  $K_I$  which are used to control the third order dynamics of the autopilot. These dynamics are due to second order dynamics and an integrator that allows the flight control system to control unstable airframe. The longitudinal autopilot design process is automated to vary the acceleration feedback loop and the pitch rate loop gains and evaluate longitudinal autopilot performance and robustness properties. The performance values examined are the normal acceleration command settling time, the percent undershoot, the percent overshoot and the steady state error.

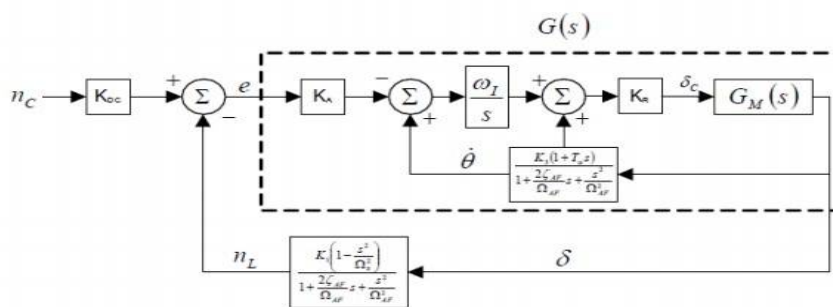


FIG. 4. Standard three-loop autopilot block diagram

The three loop autopilot, it includes an integrator for body rate in order to reduce the steady state error. It should be clear from Fig. 4 that the acceleration feedback loop is a proportional controller acting on the acceleration error. The inner loops form a proportional plus integral (PI) for pitch rate to stabilize the missile body. The outer loop relationship is given by  $e = A_{z_c}K_{DC} - A_z$  where  $A_z$  is the measured output acceleration and  $A_{z_c}$  is the input acceleration command.

Conventional “three-loop” autopilot and simple transfer function model for the actuator controller does not give acceptable performance for systems with uncertain dynamics, time delays and non-linearity [2]. Hence it is necessary to automatically tune the parameters for obtaining satisfactory response. The automatic tuning gains of

controller has been done using fuzzy logic. Based on expert knowledge a fuzzy logic system transforms a linguistic control strategy into an automatic control strategy [3]. Figure 6 shows the block diagram of a fuzzy controller. The fuzzy controller has been implemented using fuzzy logic toolbox in MATLAB.

4. USING INTELLIGENT FUZZY CONTROLLER

The fuzzy controller used in the implementation of Fuzzy longitudinal autopilot design process have fixed rule base and membership functions. The fuzzy controller has basically three main components: scaling factors, membership functions and the rules. The fuzzy controller is formed by the rule base shown as in the figure 5. The inputs to the controller are the error ( $e = A_z K_{DC} - A_z$  where  $A_z$  is the measured output acceleration and  $A_z$  is the input acceleration command.) and the rate of change of error ( $\Delta\theta$ ) while the outputs are controller gains  $K_{DC}, K_A$  and  $K_I$ .

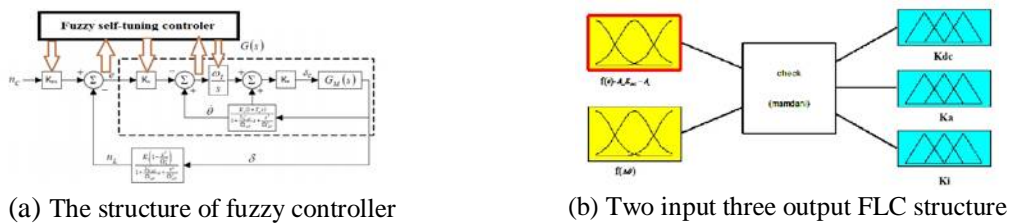


FIG. 5. Implementation of Fuzzy longitudinal autopilot design process

The structure of fuzzy controller is a two input errors (inner loops and outer loop) - three output controller gains  $K_{DC}, K_A$  and  $K_I$ . From there the range of the input as well as output membership functions have been found. The membership functions of these inputs fuzzy sets are shown in Figure 6. The linguistic variable levels are assigned as: negative big (NB), negative small (NS), zero (Z), positive small (PS) and positive big (PB). Similarly, the fuzzy set for error change  $\Delta\theta$  is presented as NB, NS, Z, PS, PB.

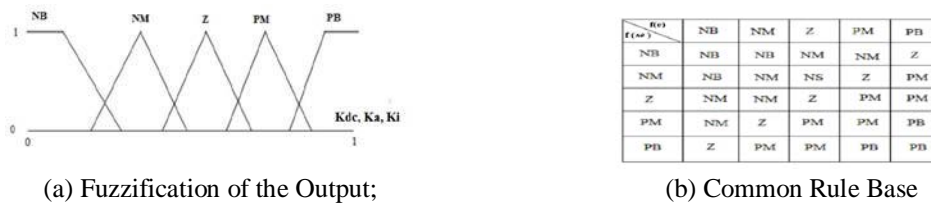


FIG. 6. Basic inference mechanism

For the output fuzzy sets the scaling of range has been done corresponding to the formulas:

$$K_{DC} = \frac{K_{DC} - K_{DCmin}}{K_{DCmax} - K_{DCmin}}; K_A = \frac{K_A - K_{Amin}}{K_{Amax} - K_{Amin}}; K_I = \frac{K_I - K_{Imin}}{K_{Imax} - K_{Imin}} \quad (6)$$

The inference mechanism has two basic tasks [7]: 1) Determining the extent to which each rule is relevant to the current situation as characterized by the two input errors - inner loops and outer loop. This task is called “matching”; 2) Drawing conclusions using the current inputs aid the information in the rule- base, this task is called “inference step”. The defuzzification phase is needed to send the rules which are evaluated in the inference phase as an unique control gains  $K_{DC}, K_A$  and  $K_I$  to the longitudinal autopilot.

## 5. SIMULATION RESULTS AND CONCLUSIONS

Simulation results for tracking system show that the fuzzy controller provides the better noise rejection as expected. Step response of the system for fuzzy application is more damped than P and PI control and control variable variations have smaller amplitudes due to the adopted defuzzification method. System setting time to reference input has an acceptable value (approximately 3.3s). The results obtained from this study has led to the further developments in the implementation of Fuzzy longitudinal autopilot design process.

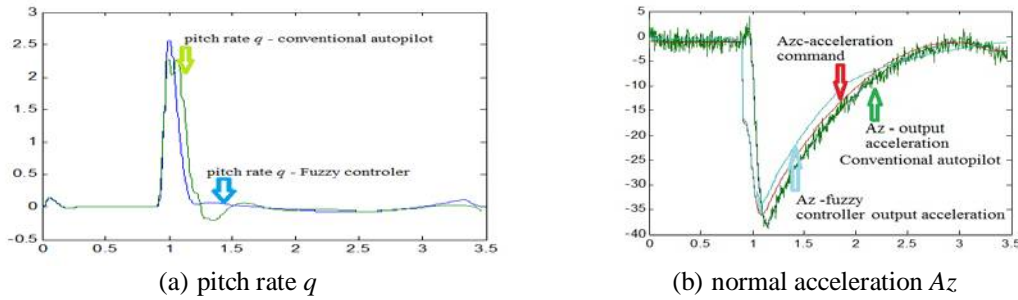
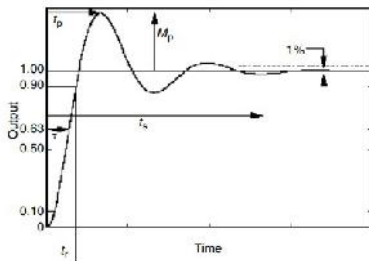


FIG. 7. Comparison among different methods for tuning longitudinal autopilot performance

The comparison among different tuning methods in terms of various performance specifications such as rise time, settling time, overshoot, undershoot and steady state error using the intelligent and conventional tuning methods has been shown in Table 1. Measure of the efficiency is how quickly the missile will respond to a change in guidance command and also the deviation of the achieved missile motion relative to the command (t is time constant,  $M_p$  is peak magnitude,  $t_p$  is time to first peak,  $t_r$  is rise time, and  $t_s$  is setting time).



(a) Time domain basic objective

| Parameters                  | Tuning Methods              |                  |
|-----------------------------|-----------------------------|------------------|
|                             | Conventional tuning methods | Fuzzy controller |
| Rise time t (sec)           | 0.545                       | 0.771            |
| Setting time $t_s$ (sec)    | 1.456                       | 1.225            |
| Overshoot $M_p$ (%)         | 12.35                       | 10.92            |
| Undershoot $M_u$ (%)        | 8.45                        | 7.35             |
| Steady state error $e_{ss}$ | 0                           | 0                |

(b) Table 1

FIG. 8. Comparison among different tuning methods in terms of various performance specifications

The various performance specifications have been improved using the intelligent method except the rise time which is less in case of conventional tuning methods. The steady state error remains zero in all the tuning methods.

GNC algorithms are diverse in type and complexity. The “tuning” process, whereby optimum values for the adjustable parameters are determined.

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