SIMPLIFIED MATHEMATICAL MODEL FOR AIRCRAFTSRESPONSE CHARACTERISTICS

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***Abstract:** Human performance modeling provides a complementary technique to develop systems and procedures that are tailored to the pilot's tasks, capabilities, limitations and also, offers a powerful technique to examine human interactions across a range of possible operating conditions. From an initial review of past efforts in cognitive modeling, it was recognized that no single modeling architecture or framework had the scope to address the full range of interacting and competing factors driving human actions in dynamic and complex environments.*

The paper presents two mathematical compensatory models, based on the aircraft dynamics characteristics and frequency responses.

***Keywords:** aircraft dynamics , Laplace transform, frequency response.*

1. INTRODUCTION

Human performance models were developed and applied to flight operations in order to predict errors and evaluate the impact of new information technologies and new procedures on flight crew performance.The usefulness of the human performance modeling to the design and evaluation of the aircraft technology is determined by the core capabilities – visual attention allocation, workload, crew interactions, procedures, situation awareness and error prediction [1]. The modeling efforts revealed that human performance models, even those cognitive architectures that have traditionally been used in the context of psychological laboratory experiments, can indeed be useful tools for complex, context dependent domains such as aviation. Specifically, the tools can be used to address the design and evaluations of aviation displays, procedures and operations.

These models can be used to inform display design and the allocation of information so as to optimize efficient scan patterns and increase the uptake of relevant information in a timely manner. Although the analysis and understanding of the airplane as an isolated unit is important, for many flight situations it is the response of the total system, made up of the human pilot and the aircraft, that must be considered. Many tasks performed by the pilots involve them in activities that resemble those of a servo control system, so, the pilot can be modeled by a set of constant coefficients linear differential equations [2, 3]. Much of research in the field of human pilot describing functions hasconcentrated on the pilot's performance in a single degree of freedom compensatory tracking task with random system inputs, where the pilot controls a single state variable through the actuation of a single control. A compensatory display is one in which the tracking error is presented, regardless of the source of error.

2. PILOT MODEL

Due to the complex nature of the situation it is possible to model the pilot in many ways and to measure the model by employing a variety of techniques. One of the most successful approaches to the measurement problem utilizes power spectral density measurements of signal circulating in the control loop. The human pilot could be replaced by a mathematical model consisting of two parts (Fig. 1): the linear describing function (written in Laplace transform notation), $Y(s)$, and the remnant, $n(t)$. Since a linear model is never able to describe the pilot completely, $Y(s)$ is insufficient by itself, and it is necessary to include the remnant $n(t)$, which is the signal that must be added in order to have all the time signals circulating in the system [4]. The $Y(s)$ selected to describe the pilot in any particular task is chosen so as to minimize that part of the input signal to the aircraft which arises from $n(t)$. Thus, the linear pilot model that results is that which accounts for as much pilot input to the aircraft as possible, and a measure of its adequacy is the fraction of the pilot input to the aircraft accounted for by $Y(s)$.
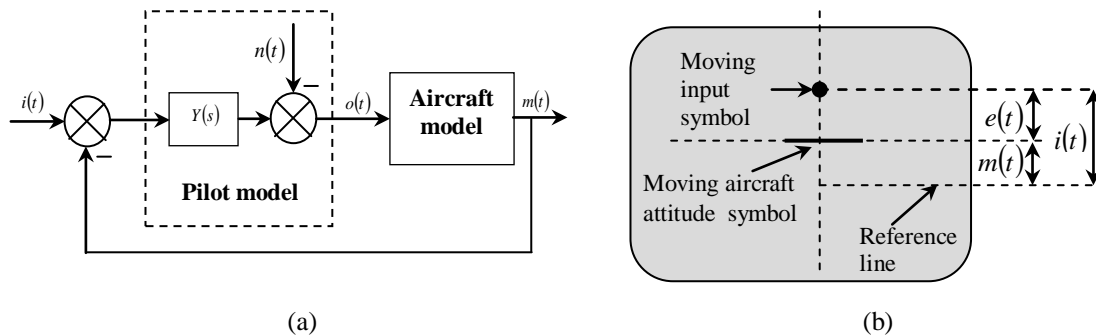

(a)   (b)

FIG. 1. Linear model of the pilot-aircraft system (a) and displayed variables (b)

According to the Fig.1, the pilot must control the aircraft response $m(t)$ in such a fashion that is matches as closely as possible the desired aircraft response. The pilot does this by viewing the instantaneous error $e(t)$ and altering his input to the aircraft (Fig. 2). It is found that the pilot's control technique is influenced by the type of input $i(t)$, the dynamics of the control system, the type of display and the dynamics of the aircraft. Although $e(t)$ is available in both cases, only the pursuit display separates the error into its components and conveys this information to the pilot. The single degree of freedom tracking task with a pursuit display is identical to the compensatory task, except that the displayed variables are different. In the compensatory task only $e(t)$ are displayed, whereas in the pursuit task both $i(t)$ and $m(t)$ are separately displayed. The data that pilots utilize to fly can be found on cockpit display panels and qualitatively different from the cues used in correspondence judgments (Fig. 2). They are data rather than cues in that they are precise, reliable indicators of whatever they are designed to represent. In the electronic environment of the cockpit, the primary task of the pilot is to supervise and monitor systems and information displays to ensure consistency of the information and to restore them when disruptions occur.

All the possible airspeed and altitude conditions of an aircraft are visualized in the flight envelope, where every aircraft performs the specific mission for which it has been designed. Within this flight envelope, desirable flying qualities are defined as a combination of characteristics both in terms of piloting the aircraft as well as in terms of the aircraft response in itself [5, 6]. Specifically, this includes the analysis of three factors: control authority (the pilot's capability to generate appropriate aerodynamic and thrust forces and moments), pilot workload (the physical effort while the pilots applies

physical forces through its arms and feet to the control commands in the cockpit of the aircraft), and pilot compensation (the mental effort by the pilot while at the controls of the aircraft).

(a)   (b)

FIG. 2. Aircraft cockpit (a) and human pilot workload (b)

The aircraft dynamical equations are nonlinear in the inertia terms and in the kinematical variables. The external force, especially the aerodynamic one, may contain inherent nonlinearities. In the automatic and powered control systems used in aerospace vehicles, there occur nonlinear control elements such as limiters, switches, and others. The human pilot is the ultimate in time-varying nonlinear systems.

3. AIRCRAFT CONTROL CHARACTERISTICS

Two control paths that the pilot employs for compensatory and precognitive control were considered. For compensatory control, the pilot observes errors between the desired and the actual response of the aircraft and applies the control to reduce or eliminate the error. If the pilot has the ability to observe the task command directly, the aircraft can be controlled to immediately follow that command without waiting for the error to develop. With a priori knowledge of the aircraft behavior, the pilot can shape these control commands to achieve the desired response.

Two models of the pursuit task are presented in fig. 3, where the pilot is represented by a pair of describing functions $(Y_1(s), Y_2(s))$ or $(Y_3(s), Y_4(s))$ since the pilot is considered to have two inputs and one output. The function $g(t)$ represents the turbulence acting on the controlled aircraft.
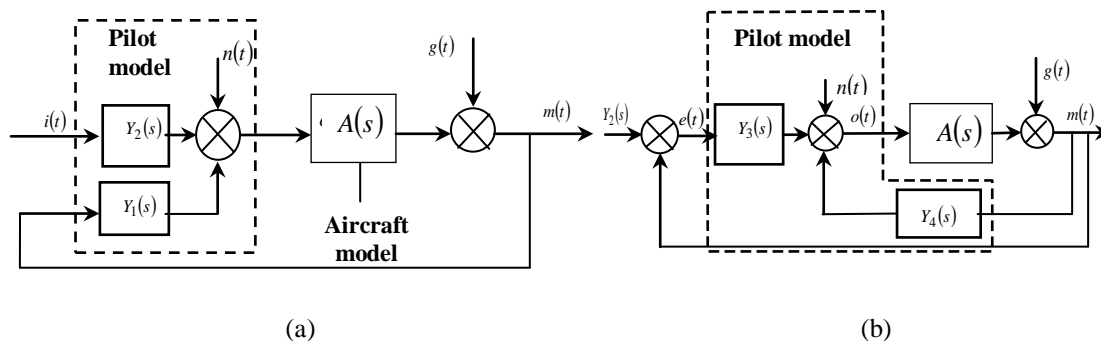

(a)   (b)

FIG. 3. Compensatory task with pilot model (a) and the model of the pursuit task (b)

The response of the aircraft to either control inputs or disturbances may be represented by the transfer function of the aircraft with respect to the controls or by the transfer

function of the aircraft with respect to the disturbances. The control input is defined both by the pilot's open-loop inputs in response to the command and the pilot closed-loop inputs in reaction to the error between the command and the aircraft's actual response.

The first element of the control system that the pilot encounters is the control inceptor, namely, stick, pedals, throttle and thrust-vector levers. Nonlinear characteristics typical of most mechanical control devices must be considered, including hysteresis, breakout force and changes in the force gradient with control deflection.

4. HIMAN PILOT MATHEMATICAL MODEL

To carry out analyses of the aircraft's flying qualities, it is necessary to have a mathematical representation of the pilot. Pilot models are typically represented in transfer function form that relate the pilot's control output in response to perceived error in the aircraft's response compared to the desired command,

$$Y_p = K_p \frac{T_L \cdot s + 1}{T_I \cdot s + 1} \cdot \frac{e^{-\tau \cdot s}}{T_P \cdot s + 1} \tag{1}$$

The first element of the transfer function is the gain, K_p, that determines the amount of control the pilot commands in proportion of the perceived error. The pilot can also perform dynamic compensation such lead (T_L - lead time constant) and lag (T_I - lag time constant), as indicated in the equation (1). Elements that cannot be adjusted are associated with the transport delay, τ, involving visual observation and mental processing of the information. The human muscle structure cannot respond instantaneously to command to move and exhibit a lag in response, which is represented by the term T_p in the equation (1). The observed variation of the T_p with forcing function bandwidth ranges from $0.1\,s$ to $0.6\,s$. The time delay represented by the $e^{-\tau \cdot s}$ term is due sensor excitation, nerve conduction, computational lags and other data processing activities in the central nervous system. The parameter τ is considered constant because it appears to be invariant with forcing function and controlled element dynamics for either single or dual random-appearing inputs tasks. The representative values for time delay are of the order of $\tau = 0.2\,s$ and the time lag T_p is approximately $0.1\,s$.

The expression $K_p (T_L \cdot s + 1)/(T_I \cdot s + 1)$ represents the major element in that adaptive capability of the pilot which allows him to control the dynamic devices. Its function is the modification of the stimulus signal into a suitable man-machine system operation.

The form of the pilot transfer function shown in the equation (1) was identified from aerospace publications and laboratory measurements acquired from human subjects. Pilot workload consists of both the mental and physical effort required to control the aircraft to achieve the desired response. Mental effort, which is difficult to quantify, is associated with anticipation required to generate lead to compensate for poor aircraft response characteristics. Physical workload can be described as the work the pilot must expend in moving the control inceptors against their resisting force. An example for an open-loop bank angle response to aircraft lateral control is

$$A(s) = \frac{L_{\delta_A}}{s(s+1)} \cdot \frac{1}{T_A \cdot s + 1} \tag{2}$$

where L_{δ_A} is the lateral control sensitivity, T_R is the roll mode time constant and T_A is the control surface actuator time constant.

For the longitudinal aircraft control, the simplified system of equations has the form

$$\begin{bmatrix} s \cdot U_1 - Z_\alpha & -s \cdot U_1 \\ -\left(M_{\dot\alpha}s + M_\alpha\right) & s^2 - sM_q \end{bmatrix} \cdot \begin{bmatrix} \alpha(s) \\ \theta(s) \end{bmatrix} = \begin{bmatrix} Z_{\delta_e} \\ M_{\delta_e} \end{bmatrix} \delta_e(s) \tag{3}$$

where s is the Laplace transform parameter. The other variables in the above equation are the longitudinal control derivatives, the pitch and attack angles (θ and α) and the elevator command δ_e [7, 8].

In order to obtain the dynamic response characteristics of the human pilot, we have made laboratory measurements acquired from human subjects, for control forces of the stick, pedals, throttle and aircraft command surfaces. We have estimated the time delay, the lead and lag time constants of the pilot model represented in transfer function form and also we built up the mathematical model of the whole system "pilot and aircraft".

5. NUMERICAL RESULTS

The impulse and step responses are calculated for an aircraft with the following parameters: wing span, $8\,m$; mean aerodynamic chord, $1.5\,m$; sweep angle, $18°$; mass of aircraft $3000\,kg$; I_{xx} moment of inertia, $1000\,kg \cdot m^2$; I_{yy} moment of inertia, $6200\,kg \cdot m^2$; I_{zz} moment of inertia, $6800\,kg \cdot m^2$; I_{xz} product of inertia, $200\,kg \cdot m^2$; flight Mach number, 0.7. Some numerical results are presented in the following figures.
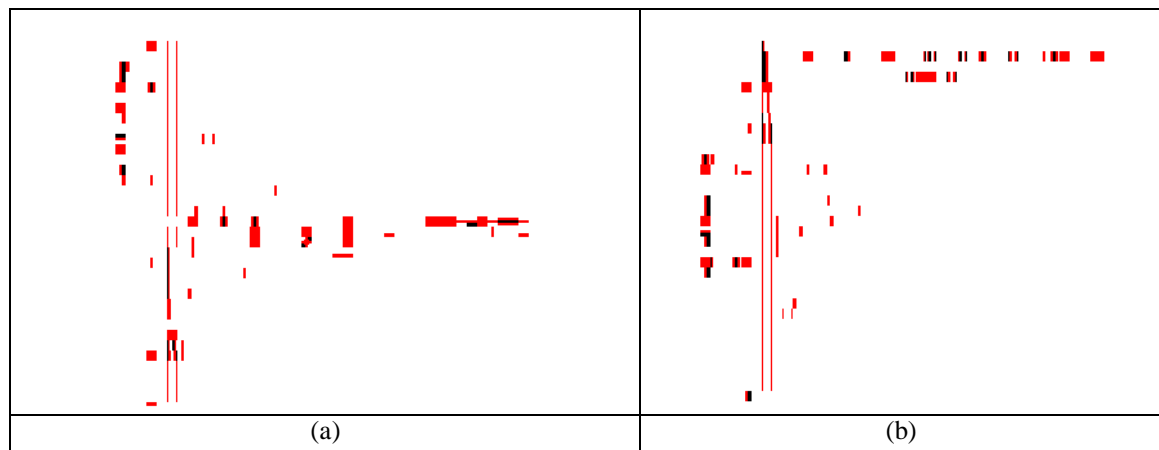

| (a) | (b) |

FIG. 4. Impulse response (a) and step response (b) for the aircraft elevator command
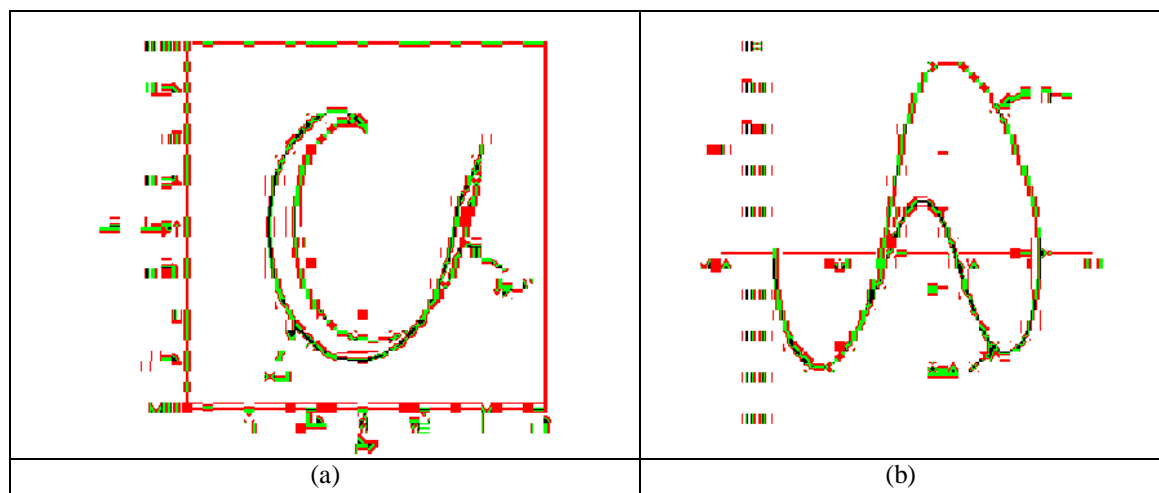

| (a) | (b) |

FIG. 5. Polar plot of pilot-aircraft transfer function in lateral (a) and longitudinal (b) movements

The dynamic characteristics of the integrated "pilot-aircraft" system were studied with the frequency-response method. The middle line (red color) in fig. 5 corresponds to the following parameters: $K_p = 1$; $T_L = 2$; $T_P = 0.5$; $\tau = 0.15$.

CONCLUSIONS

The assessment of handling or flying qualities of airplane depends on pilot skills. When the pilot flies an aircraft he forms subjective opinions concerning the suitability of the man-machine system for performing the assigned task. In arriving at an assessment he is influenced by many parameters. These range over a wide spectrum and include the response to external disturbances, the ease with which instruments can be read, mission, visibility, weather conditions and the familiarity of the pilot with the present aircraft and mission. To be able to asses aircraft handling qualities and human performances, one must have a measuring technique with which any given aircraft characteristics can be rated. If the pilot is expected to assume manual control the system should be structured that he is either kept actively in the control loop at all times or is constantly made aware of the feel of the present aircraft configuration through some auxiliary task which he can practice on during critical phase of the flight.

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