DESIGN AND DEVELOPMENT OF THE LQR OPTIMAL CONTROLLER FOR THE UNMANNED AERIAL VEHICLE

Róbert SZABOLCSI

Óbuda University, Budapest, Hungary (szabolcsi.robert@bgk.uni-obuda.hu)

DOI: 10.19062/1842-9238.2018.16.1.7

Abstract: *This study deals with the optimal control of unmanned aerial vehicles (UAVs). Concerns about the minimum energy consumption of the UAV are still in the focus of attention of many researchers. The optimal control based on the cost function minimization of the closed loop automatic flight control systems of the UAV is one of the techniques effectively supporting solution of the gain selection of UAV autopilots. Recently, the worldwide application of the UAVs has resulted in the wider application of the computer aided design of the closed loop flight control systems. This study highlights the optimal control of the multivariable UAV control systems, and presents a new design example based on Linear Quadratic Regulator (LQR) optimal control.*

Keywords: *UAV, flight control system, optimal control, LQR design.*

1. INTRODUCTION

The optimal control has a long history. The LQR optimal design technique is still continuing to gain popularity among the optimal design methods currently available. The basic idea behind this method is that control law is designed via the minimization of the pre-defined quadratic integral performance criteria. The dynamical system being considered like UAV dynamics is a deterministic one, so latter work will extend the challenge of the controller design to the random systems. Solution of such design programs is supported often by such computer software as MATLAB®. In this paper, the author will present the solution of the basic mathematical problem using calculus of variations, like solution of the matrix algebraic Ricatti equation (MARE). This method gained degraded importance in modern control engineering. Finally, a design example will demonstrate a numerical example for the solution of the LQR design problem. A unique principle of setting weighting matrices in quadratic integral criteria using unit weights with further heuristic scheduling of weights will be presented.

2. LITERATURE REVIEW

Integral performance indices have been exhaustively demonstrated in [1, 10, 11, 12, 15]. There is a large variety of UAVs, being investigated and demonstrated in control law synthesis meaning. Design of the multirotor UAV, say, tri-, or quad copters are demonstrated in [2, 3, 4, 5]. The fixed-wing UAVs autopilot design examples are duly demonstrated by [6, 7, 8, 9]. The application of the LQR design method applied for finding optimal control laws is elaborated in works [3, 4, 5, 6, 7, 8, 9]. UAV automatic flight control systems design requirements are elaborated and presented in [13, 14]. The solution of the LQR design problem will be supported by MATLAB [16] and Control System Toolbox [17].

The impressive development path of the UAVs segmented to that of the classical and modern era is outlined in [18]. The challenging problem of the UAV integration into air defense is evaluated and a certain solution is proposed in [19].

3. LINEAR QUADRATIC PERFORMANCE CRITERIA

The optimal design of the closed loop control systems is a well-known design technique of the multivariable (MIMO) dynamic systems [1, 10, 11, 12, 15]. Optimal controllers, say, full state feedback gain matrix **K** is designed and scheduled to minimize the performance index describing the cost function of the system. Let us consider the multivariable deterministic system, and, it is also supposed that all *n* state variables are measurable ones and available for the controller. The state and output equations can be given as follows below [1, 11, 12, 15, 16, 17]:

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + \mathbf{B}\boldsymbol{u};\ \boldsymbol{y} = \mathbf{C}\boldsymbol{x} + \mathbf{D}\boldsymbol{u} \tag{1}$$

where **x** is a column state vector of length *n*, **u** is the control input vector of length *r*, **A** is an ($n\times n$) square state matrix; **B** is an ($n\times r$) input matrix; **y** is a column output vector; **C** is an ($m\times n$) the output matrix; and finally, **D** is an ($m\times r$) direct feedforward matrix.

For many physical systems the matrix, **D** is a null matrix. Thus, the system state and output equations can be represented in the following notation:

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + \mathbf{B}\boldsymbol{u};\ \boldsymbol{y} = \mathbf{C}\boldsymbol{x} \tag{2}$$

Block diagram of the open loop UAV dynamics built by equation (2) can be seen in Fig.1.

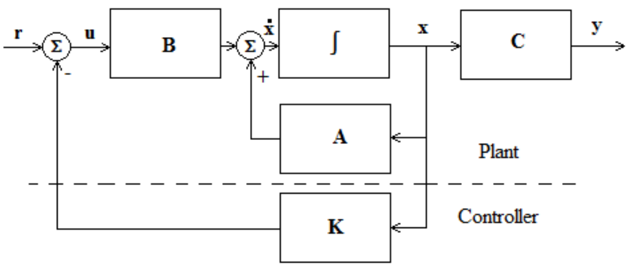


FIG. 1. Closed Loop block diagram.

The control law can be expressed using state feedback gain matrix of **K**, thus:

$$\dot{\boldsymbol{x}} = [\mathbf{A} - \mathbf{BK}]\boldsymbol{x} + \boldsymbol{BKr} = \mathbf{A}_{cl}\boldsymbol{x} + \mathbf{B}_{cl}\boldsymbol{r} \tag{3}$$

To find optimal control law, i.e. optimal state feedback gain matrix $\mathbf{K}_{opt}$ for zero reference signal, $\boldsymbol{r}$(t)=0, first let us find criteria of optimality. Let us consider a dynamical system with fixed end time, t_f. Let formulate the control problem: choose control vector $\boldsymbol{u}$(t) such that it minimizes the following cost function [1, 10, 11, 12, 15, 16, 17]:

$$J = \psi\left(\boldsymbol{x}(t_f)\right) + \int_{t_0}^{t_f} L(\boldsymbol{x}(t), \boldsymbol{u}(t), t)dt \rightarrow Min \tag{4}$$

subject to $\dot{\boldsymbol{x}} = f(\boldsymbol{x}(t), \boldsymbol{u}\ (t), t)$ (5)

with initial conditions of $\boldsymbol{x}(t_0) = \boldsymbol{x}_0$ (6)

where J is the total cost, $\psi\left(\boldsymbol{x}(t_f)\right)$ is the terminal cost.

It I supposed that $L(\boldsymbol{x}(t), \boldsymbol{u}(t), t)$ is the non-negative cost function. Let us augment the cost function of (4) with co-state vector of $\boldsymbol{\lambda}(t)$ [10, 11]. The augmented total cost function now is as follows:

$$\bar{J} = \psi\left(\boldsymbol{x}(t_f)\right) + \int_{t_0}^{t_f} \left(L + \boldsymbol{\lambda}^T (f - \dot{\boldsymbol{x}}) \right) dt \rightarrow Min \tag{7}$$

The function of $\boldsymbol{\lambda}(t)$ can be chosen to be of any mathematical form, because it multiplies term of $f - \dot{\boldsymbol{x}} = 0$. It is well-known that along the optimal trajectory variations both in J and $\bar{J}$ should die as $t \rightarrow t_f$. Variation of the augmented cost function of (7) can be derived as given below:

$$\delta \bar{J} = \psi_x \delta \boldsymbol{x}(t_f) + \int_{t_0}^{t_f} \left(L_x \delta \boldsymbol{x} + L_u \delta \boldsymbol{u} + \boldsymbol{\lambda}^T f_x \delta \boldsymbol{x} + \boldsymbol{\lambda}^T f_u \delta \boldsymbol{u} - \boldsymbol{\lambda}^T \delta \dot{\boldsymbol{x}} \right) dt \tag{8}$$

where $\psi_x = \frac{\partial \psi}{\partial x}$, $L_x = \frac{\partial L}{\partial x}$, $L_u = \frac{\partial L}{\partial u}$, $f_x = \frac{\partial f}{\partial x}$, $f_u = \frac{\partial f}{\partial u}$.

Integrating by parts, the last term of the integrand of equation (8) can be expressed in the following form:

$$-\int_{t_0}^{t_f} \boldsymbol{\lambda}^T \delta \dot{\boldsymbol{x}} dt = -\boldsymbol{\lambda}^T (t_f) \delta \boldsymbol{x}(t_f) + \boldsymbol{\lambda}^T (t_0) \delta \boldsymbol{x}(t_0) + \int_{t_0}^{t_f} \dot{\boldsymbol{\lambda}}^T \delta \boldsymbol{x} dt \tag{9}$$

Substituting equation (9) into equation (8) yields to the following augmented cost function:

$$\delta \bar{J} = \psi_x \left(\boldsymbol{x}(t_f)\right) \delta \boldsymbol{x}(t_f) + \int_{t_0}^{t_f} (L_u + \boldsymbol{\lambda}^T f_u) \delta \boldsymbol{u} dt + \int_{t_0}^{t_f} \left(L_x + \boldsymbol{\lambda}^T f_x + \dot{\boldsymbol{\lambda}}^T \right) \delta \boldsymbol{x} dt - \boldsymbol{\lambda}^T (t_f) \delta \boldsymbol{x}(t_f) + \boldsymbol{\lambda}^T (t_0) \delta \boldsymbol{x}(t_0) \tag{10}$$

Initial conditions can't vary at a later time. Thus, the last term in equation (10) is equal to zero. By evaluating the augmented cost function $\bar{J}$ defined by equation (10), it becomes evident that there are three variations inside the equation, which must be independently zero, i.e. any of $\boldsymbol{x}$(t), $\boldsymbol{u}$(t), or $\boldsymbol{x}(t_f)$ can be varied:

$$L_u + \boldsymbol{\lambda}^T f_u = 0 \tag{11}$$

$$L_x + \boldsymbol{\lambda}^T f_x + \dot{\boldsymbol{\lambda}}^T = 0 \tag{12}$$

$$\psi_x\left(\boldsymbol{x}(t_f)\right) - \boldsymbol{\lambda}^T (t_f) = 0 \tag{13}$$

Re-arranging equations (12) and (13) yields to:

$$-L_x - \boldsymbol{\lambda}^T f_x = \dot{\boldsymbol{\lambda}}^T \tag{14}$$

$$\psi_x\left(\boldsymbol{x}(t_f)\right) = \boldsymbol{\lambda}^T (t_f) \tag{15}$$

The primary difficulty of the solution of that kind of optimal control problem is that state variables of the dynamical system propagate forward, while the co-state equation propagates backwards. The evolution of the co-state vector $\boldsymbol{\lambda}(t)$ is represented in reverse time, from its final state to the initial state. Next chapters deal with the solution of the optimization problems in backward time.

3.1 Solution of the optimal design problems using gradient method. The numerical solutions of the optimal control problems using gradient method can be explained in the following iterative steps and loops [10, 11, 12, 15, 16, 17].

Step 1) Define control input $\boldsymbol{u}$(t), for the given $\boldsymbol{x}_0$.

Step 2) To create the state trajectory, propagate state equation of $\dot{\boldsymbol{x}} = f(\boldsymbol{x}(t), \boldsymbol{u}\ (t), t)$ forward in time.

Step 3) Evaluate terminal cost function of $\psi_x\left(\boldsymbol{x}(t_f)\right)$, and propagate co-state vector of $\boldsymbol{\lambda}(t)$ backward in time, from t_f to t_0 using equation (14).

Step 4) At each step choose for the control input variation the following formula: $\delta u = -\mathbf{K}(L_u + \boldsymbol{\lambda}^T f_u)$, where **K** is positive scalar, or, for multi input systems, positive definite matrix.

Step 5) Letting $u = u + \delta u$.

Step6) Go back to Step 2, and repeat the calculation loop until solution converges.

3.2 The LQR solution of the optimal control design problem. Let us set terminal cost at zero, i.e. $\psi = 0$, and let the cost function L be defined as follows [10, 11, 12]:

$$L = \frac{1}{2}\boldsymbol{x}'\boldsymbol{Q}\boldsymbol{x} + \frac{1}{2}\boldsymbol{u}'\boldsymbol{R}\boldsymbol{u} \tag{16}$$

where, $L \geq 0$, $\boldsymbol{Q} \geq 0$ weighting matrix, $\boldsymbol{R} > 0$ weighting matrix.

For the linear (rather linearized) dynamical systems, one can set following equations:

$$L_x = \boldsymbol{x}'\boldsymbol{Q} \tag{17}$$

$$L_u = \boldsymbol{u}'\boldsymbol{R} \tag{18}$$

$$f_x = \mathbf{A} \tag{19}$$

$$f_u = \mathbf{B} \tag{20}$$

so that we have:

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + \mathbf{B}\boldsymbol{u} \tag{21}$$

$$\boldsymbol{x}(t_0) = \boldsymbol{x}_0 \tag{22}$$

$$\boldsymbol{\lambda} = -\mathbf{Q}\boldsymbol{x} - \boldsymbol{A}'\boldsymbol{\lambda} \tag{23}$$

$$\boldsymbol{\lambda}(t_f) = 0 \tag{24}$$

$$\boldsymbol{R}\boldsymbol{u} + \boldsymbol{B}'\boldsymbol{\lambda} = 0 \tag{25}$$

Being interested in linear dynamical systems, the co-state vector can be represented as $\boldsymbol{\lambda} = \boldsymbol{P}\boldsymbol{x}$, where $\boldsymbol{P}$ is the cost matrix. By substituting this equation into equation (23), and using equation (21), we can get the following matrix-differential equation [10, 11, 12, 15, 16, 17]:

$$\boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}'\mathbf{P} + \mathbf{Q} - \mathbf{P}\mathbf{B}\boldsymbol{R}^{-1}\boldsymbol{B}'\mathbf{P} + \boldsymbol{P} = 0 \tag{26}$$

Equation (26) is the matrix Ricatti equation (MRE). If $t_f \rightarrow \infty$, and $\boldsymbol{Q}$=const, and $\boldsymbol{R}$=const, $\dot{\boldsymbol{P}} \rightarrow 0,\ \forall t$, i.e. the steady-state solution of the equation (26) can be rewritten as follows [10]:

$$\boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}'\mathbf{P} + \mathbf{Q} - \mathbf{P}\mathbf{B}\boldsymbol{R}^{-1}\boldsymbol{B}'\boldsymbol{P} = 0 \tag{27}$$

Solution of the equation (27) called the matrix algebraic Ricatti equation (MARE) yields to the cost matrix $\boldsymbol{P}$. Finding solution to the MARE is supported by many numerical tools in linear algebra.

MATLAB supports solution of Ricatti equations both in continuous (*are.m*) and in discrete time domain (*dare.m*) [16, 17]. Finally, equation $\boldsymbol{Ru} + \boldsymbol{B}^T\boldsymbol{\lambda} = 0$ will determine the optimal feedback law as it given below [10, 11, 12, 15, 16, 17]:

$$\boldsymbol{u} = -\boldsymbol{R}^{-1}\boldsymbol{B}^{\prime}\boldsymbol{Px} = -\mathbf{K}\boldsymbol{x} \tag{28}$$

where $\mathbf{K}_{opt} = \boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}$ is the optimal state-feedback gain matrix for multivariable dynamical systems, or optimal scalar gain.

The optimal controller synthesis includes following steps [1, 10, 11, 12, 16, 17]:

Step 1) The pair $\{\mathbf{A}, \mathbf{B}\}$ must be controllable, and the pair $\{\mathbf{A}, \mathbf{C}\}$ must be observable by R. Kalman.

Step 2) Define weighting matrices of **Q** and **R** by Bryson's Rule.

Step 3) Solve MARE (equation 27) to find cost matrix of ***P***.

Step 4) Substitute matrix ***P*** into equation (28) to find optimal control law.

Step 5) Check closed loop dynamic performances for similarity with those of the pre-defined ones.

Step 6) If there is no precise match with the required performances, return to Step 2 and change weights heuristically whilst dynamic performances are met.

4. DESIGN OF THE LQR OPTIMAL CONTROLLER FOR THE SMALL UAV

The identified dynamical model of the short period lateral/directional motion of the Boomerang-60 Trainer UAV can be derived as follows below [15]:

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu} = \begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -0,7724 & 0 & -18,9671 & 9,0867 \\ 1,9247 & -19,9149 & 7,7565 & 0 \\ 69,1314 & -23,8689 & -2,5966 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + + \begin{bmatrix} 0 & 2,2582 \\ -23,8289 & 1,5015 \\ -11,7532 & -15,2855 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \tag{29}$$

where v is the lateral speed, p is the roll rate, r is the yaw rate, ϕ is the roll angle position, δ_a is the angular deflection of the ailerons, and, finally, δ_r is the change in rudder angular position.

Let us find stabilizing LQR controller of the Boomerang-60 Trainer UAV able to manipulate short period motion of the roll position angle. Prior to any kind of design implemented, the dynamical model of the UAV defined by equation (29) must be reduced to that of the short period one. One can get the following state space model:

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu} = \begin{bmatrix} \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -19,9149 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} p \\ \phi \end{bmatrix} + \begin{bmatrix} -23,8289 \\ 0 \end{bmatrix} \delta_a \tag{30}$$

By using the $\mathbf{A}, \mathbf{B}$ pair of matrices, the system's controllability has been evaluated. The controllability matrix was calculated to be [16, 17]:

$$\boldsymbol{Co} = \begin{bmatrix} -23.8289 & 474.5502 \\ 0 & -23.8289 \end{bmatrix} \tag{31}$$

which has a rank of 2, i.e. the dynamical system is the controllable one using Kalman-criteria.

By using the $\mathbf{A}, \mathbf{C}$ pair of matrices, the observability matrix has been calculated to be [16, 17]:

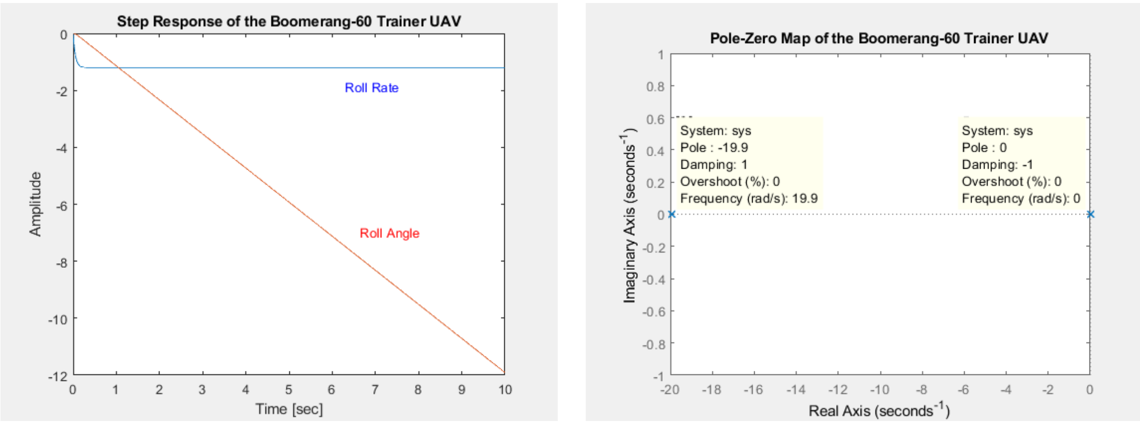
$$\boldsymbol{Ob} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -19.9149 & 0 \\ 1 & 0 \end{bmatrix} \qquad (32)$$

which has a rank of 2, i.e. the dynamical system is the observable one using Kalman-criteria.

The time domain behavior of the lateral short period motion of the UAV has been analyzed. The result of the computer simulation can be seen in Fig. 2.a. The input of the UAV was the unit step change in the aileron angular position, i.e. $\delta_a = 1(t)\ deg$.

Fig. 2.a represents the roll rate and the roll angle behavior of the lateral motion of the UAV. The roll rate behaves as an exponential function, while the roll angle is an integral of the roll rate, i.e. it is a monotone increasing function of time.

The open loop UAV has two poles on the complex plain. The poles and dynamic performances can be seen in Fig. 2.b. From these s-plane roots it is easy to see that aperiodic instability can be eliminated using full state feedback, and the design procedure implemented will ensure the optimal solution.



(a) (b)

FIG. 2. The open loop system behavior of the UAV (MATLAB-script: the author).

The UAV closed loop system is supposed to exclude oscillatory behavior, and the dynamic performance expressed in settling time of the closed loop control system used for the design goal was [14]:

$$t_s \le 2\ sec \qquad (33)$$

During controller design, weighting matrices for the first trial have been chosen as follows:

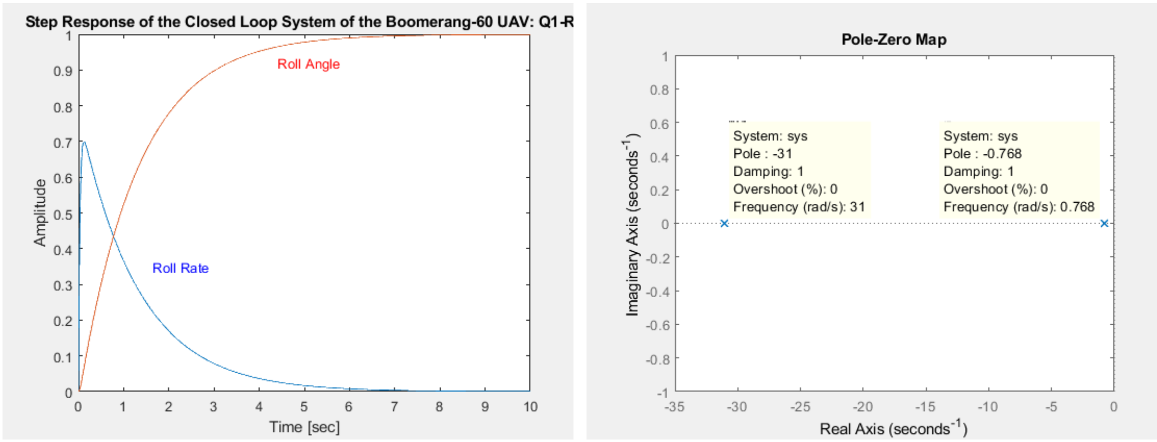
$$\boldsymbol{Q}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};\ \boldsymbol{R}_1 \qquad (34)$$

The dynamical system is observable and controllable by the Kalman-criteria, thus, the optimal controller can be designed. The optimal controller was designed using the MATLAB *lqr2.m* function. Using equation (34), the cost matrix of $\boldsymbol{P}$, as solution of the MARE, and optimal state feedback gain matrix of **K** have been calculated to be [16, 17]:

$$\boldsymbol{K}_1 = [-0{,}4993 \quad -1{,}0000];\ \boldsymbol{P}_1 = \begin{bmatrix} 0{,}0210 & 0{,}0420 \\ 0{,}0420 & 1{,}3351 \end{bmatrix} \tag{35}$$

The closed loop UAV system has been evaluated in time domain. The closed loop system response was found for the unit step change of the roll angle, i.e. $\phi_{ref} = 1(t)\ deg$. Fig. 3.a. represents roll rate and roll angle time domain behavior. Finding settling time for the 5% static tolerance field yields to $t_s \cong 4\ sec$, which represents a very slow behavior of the UAV.

Roots of the closed loop control system of the UAV are located at $p_1 = -31$, and $p_2 = -0{,}768$. Thus, the open loop system root from the origin of the *s*-plane was shifted to that of the new coordinate of $p_2 = -0{,}768$.(see Fig. 3. b.). In other words, the state feedback was used to ensure the stability of the closed loop control system of the UAV.



(a) (b)

FIG. 3. The closed loop control system behavior of the UAV (MATLAB-script: the author).

From Fig. 3.a. it is easy to determine that the closed system time domain behavior is too slow. Therefore, to accelerate the transient response, let us use for the controller synthesis the following weighting matrices set heuristically to be:

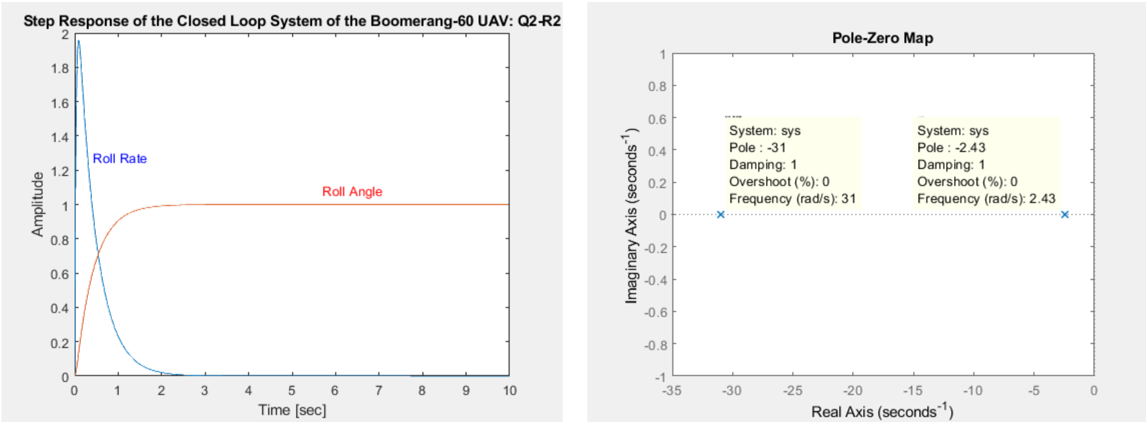
$$\boldsymbol{Q}_2 = \begin{bmatrix} 1 & 1 \\ 0 & 10 \end{bmatrix};\ \boldsymbol{R}_2 = 1 \tag{36}$$

The optimal controller was synthesized using the *lqr2.m* function of MATLAB. Using equation (36), the cost matrix of ***P***, and optimal state feedback gain matrix of **K** have been calculated to be [17]:

$$\boldsymbol{K}_1 = [-0.5656 \quad -3.1623];\ \boldsymbol{P}_1 = \begin{bmatrix} 0.0237 & 0.1327 \\ 0.1327 & 4.4316 \end{bmatrix} \tag{37}$$

The UAV time domain behavior has been evaluated. Results of the computer simulation can be seen in Fig. 4.

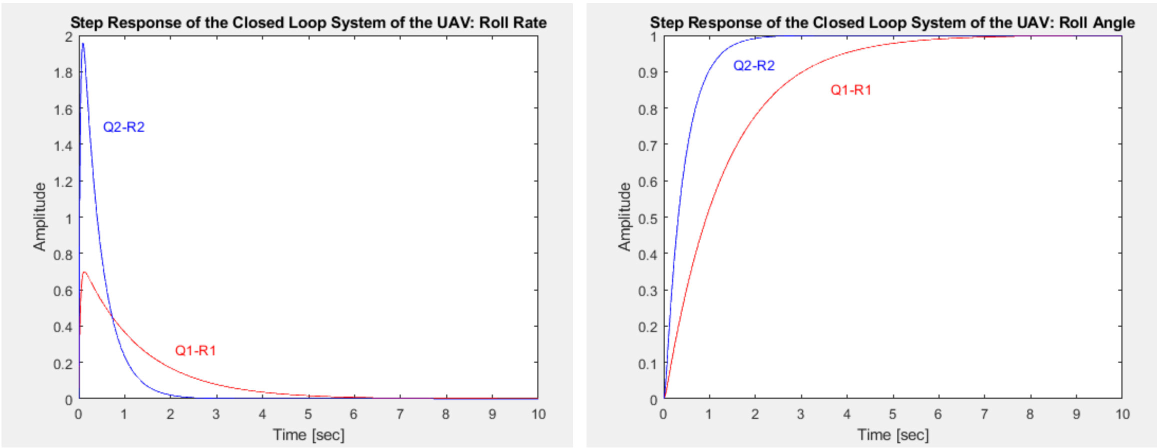
Fig. 4.a. demonstrates that the closed loop system of the UAV has faster response to the reference input. Finding settling time for the 5% static tolerance field yields to $t_s \cong 1{,}5\ sec$, which is in line with the criteria defined by equation (33). In Fig. 4.b. it is easy to see that a pole of $p_2 = -0{,}768$ is shifted to the newest place on the *s*-plane determined by $p_2 = -2{,}43$, whilst position of the pole with coordinate of $p_1 = -31$ is not varied.



(a) (b)

FIG. 4. The closed loop control system behavior of the UAV (MATLAB-script: the author).

The UAV's closed loop systems step responses have been compared. Results of the computer simulation can be seen in Fig. 5.



(a) (b)

FIG. 5. The UAV's closed loop control system behavior with different weights (MATLAB-script: the author).

Fig. 5.b. shows the UAV's roll angle outer loop of the closed loop control system. The heuristic set of the weighting parameters of **Q** and **R** has led to the system response with pre-defined dynamic performances given by equation (33). The roll rate inner loop transient can be seen in Fig. 5.a., which represents a meaningful increase of the maximum value of the roll rate. If such change in the roll rate amplitude is not allowed, weighting matrices of the integral performance index **Q** and **R** must be changed to those which would ensure a more complex and sophisticated set of closed loop dynamic performances.

The heuristic change of the weighting matrices requires high-level engineering experiences deduced from the solution of different problems of modern control engineering and optimal control. Moreover, the engineering intuitions can help scheduling the process described above.

5. CONCLUSIONS

The reason behind this research was to solve the basic optimization LQR design problem. This study has presented the solution to the LQR design method using calculus of variations. The optimal control strategy implemented for the design of the deterministic dynamical systems like UAV spatial motion has kept importance till recent days.

The study has provided striking facts regarding the optimal settings in the closed loop flight control systems of the unmanned aerial vehicles, which is an emerging problem during the flight path design of the UAVs, extending flight radius, or flight time. The proposed method and the design example presented in this paper is the first step in the solution of more complex and challenging engineering design problems.

Next step following the LQR design stage elaborated in this paper is the evaluation of the fitness of the proposed solution to the more sophisticated set of dynamic performance criteria. If there is a lack of any dynamic performances, the static proportional controller of the LQR solution will be supplemented with an integral term, so as to improve disturbance rejection ability. If it leads to extended settling time, a derivative term also must be introduced. The augmentation of the proposed results and future work is about to apply optimal PID-controllers ensuring dynamic performances set prior to.

REFERENCES

[1] J. Bokor, P. Gáspár, Z. Szabó, *Modern Control Engineering*, Budapest, Budapest University of Technology and Economics, 2014.

[2] Z. Tahir, M. Jamil, S. A. Liaqat, L. Mubarak, Waleed Tahir and Syed Omer Gilani, Design and Development of Optimal Control System for Quad Copter UAV, *Indian Journal of Science and Technology*, Vol 9(25), 10.17485/ijst/2016/v9i25/96611, July 2016.

[3] C. Z. Myint, H. M. Tun, Z. M. Naing, Development Of Linear Quadratic Regulator Design For Small UAV System; *International Journal Of Scientific & Technology Research* Volume 5, Issue 07, July 2016.

[4] A. Joukhadar, I. Hasan, A. Alsabbagh, M Alkouzbary, Integral Lqr-Based 6dof Autonomous Quadrocopter Balancing System Control, *(IJARAI) International Journal of Advanced Research in Artificial Intelligence*, Vol. 4, No.5, 2015

[5] D. C. Tosun, Y. Işık, H. Korul, Comparison of PID and LQR controllers on a quadrotor helicopter, *International Journal of Systems Applications, Engineering & Development*, Volume 9, 2015.

[6] K. Turkoglu, E. M. Jafarov, *Augmented optimal LQR control system design for the longitudinal flight dynamics of an UAV: Inner and outer loop concepts*, Proceedings of the 9th WSEAS International Conference on Automatic Control, Modeling & Simulation, Istanbul, Turkey, May 27-29, 2007.

[7] K. K. Yit, P. Rajendran, L. K. Wee, Proportional-derivative linear quadratic regulator controller design for improved longitudinal motion control of unmanned aerial vehicles, *International Journal of Micro Air Vehicles* January-March 2016: 41–50.

[8] C. Hajiyev, S. Y. Vural, LQR Controller with Kalman Estimator Applied to UAV Longitudinal Dynamics, *SciRes Journal*, 2013, 4, 36-41.

[9] H. Purnawan, Mardlijah, E. B. Purwanto, Design of linear quadratic regulator (LQR) control system for flight stability of LSU-05, *IOP Conf. Series: Journal of Physics: Conf. Series* 890 (2017) 012056.

[10] K. Ogata, *Modern Control Engineering*, New York – London, Prentice-Hall, 1999.

[11] D. McLean, *Automatic Flight Control Systems*. New York – London – Toronto – Sydney – Tokyo – Singapore. Prentice-Hall International Ltd., 1990.

[12] R. Szabolcsi, *Computer Aided Design of Modern Control Systems*, Budapest, Miklós Zrínyi National Defense University, 2011.

[13] R. Szabolcsi, Longitudinal Motion Flying Qualities Applied in Airworthiness Certification Procedure, *Land Forces Academy Review* 74:(2), 208-216, 2014.

[14] R. Szabolcsi, Lateral/Directional Flying Qualities Applied in UAV Airworthiness Certification Process, *Land Forces Academy Review*, 3/2014:(75) pp. 336-346, 2014.

[15] R. Szabolcsi, *Automatic Flight Control of the* UAV, Budapest, Óbuda University, 2016.

[16] MATLAB R2017b, User's Guide, The MathWorks, 2017.

[17] MATLAB Control System Designer/Control System Toolbox 10.3, User's Guide, The MathWorks, 2017.
[18] S. Pop, A. Luchian, R. G. Zmădu, E. Olea: The Evolution of Unmanned Aerial Vehicles. *Review of the Air Force Academy*, No.3 (35)/2017, pp(125-132), 2017.
[19] V. Șandru, M. Rădulescu: The Use of UAV's During Actions of Integrated Air Defense Systems. *Review of the Air Force Academy*, No 3 (30) 2015, pp (133-138), 2015.