TRACKING MANEUVERING TARGET WITH KALMAN FILTER AND INTERACTIVE MULTIPLE MODEL

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Abstract: *In this work, a study has been conducted on the capabilities of the Kalman and Interactive Multiple Model filters to track the trajectory of a maneuvering target. The results of the experiment clearly show the better performance of the Multiple Model algorithm than Kalman's.*

Keywords: *tracking, Kalman filter, Interactive Multiple Model*

1. INTORDUCTION

The most effective solution that does not require the deployment of new facilities in radar systems to make better Air Traffic Control (ATC) is to improve the algorithms it operates with. The main challenge and difficulty of ATC is to track a maneuvering target in a clutter environment [3]. The present work explores existing algorithms for processing radar information. One is the standard linear Kalman filter widely used in tracking systems, and the other is the Bar-Shalom tracking algorithm called the Interactive Multiple Model (IMM) algorithm [4] that estimates with significant noise suppression and rapid sequence response from target maneuvers. The paper compares the capabilities of the two algorithms with regard to the accuracy in tracking the target trajectory.

2. KALMAN FILTER

It is a repeating mathematical process that uses certain equations and sequential input data to quickly calculate the true values of the object to be measured when the measured values contain unpredictable or random errors.

An important advantage of the Kalman filter is that the obtained theoretical solution directly determines its practical realization and the expressions for assessing the state vector and its covariance have a recurrent form which provides a sequential refinement of the state vector upon receipt of each new measurement. It is assumed that all trajectory information is locked in the last estimate, so it is not necessary to store all previous measurements and process their entire set after receiving the new measurement [5].

Kalman filter - this is a recursive Bayesian algorithm for optimal linear filtration of the vector random process. The algorithm (Fig.1) is the solution of the following equation system [2].

$$
\begin{aligned}
&z_k = H\,x_k + v_k\\
&x_k^- = F\,\hat{x}_{k-1} + w_k\\
&P_k^- = F\,\hat{P}_{k-1}F^T + Q\\
&S_k = H\,P_k^- H^T + R\\
&K_k = P_k^- H^T S_k^{-1}\\
&\hat{x}_k = x_k^- + K_k\left(z_k - H\,x_k^-\right)\\
&\hat{P}_k = \left(I - K_k H\right)P_k^-
\end{aligned}
\qquad (1)
$$

The task of Kalman filter is to find an optimal estimation of the *k* stage vector state of its covariate matrix $\hat{p}_k$. Matrices **F**, **H**, **Q** and **R** are considered to be known.

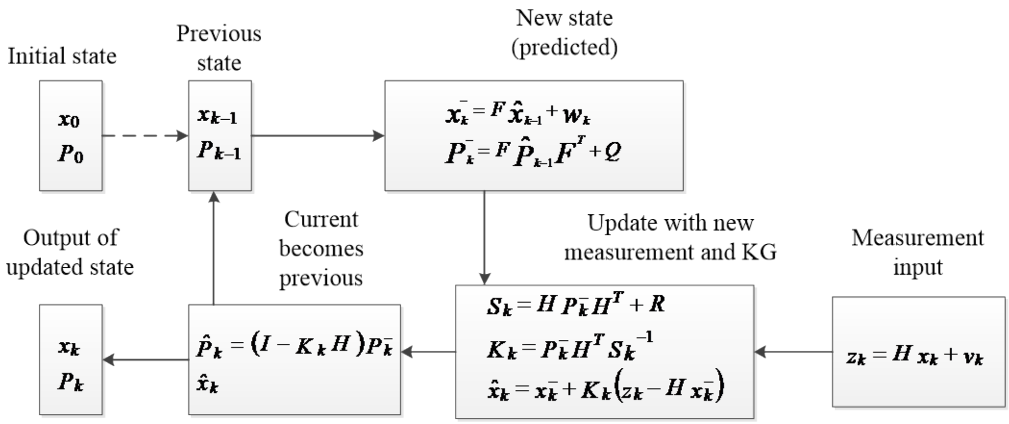

FIG. 1 A complete picture of Kalman filter operations

According to the summarized Bayesian estimating scheme, the Kalman filter consists of two sequentially coupled devices, an extrapolator predicting the state of the object step by step, and a filter specifying the extrapolated meaning based on the new measurement. The extrapolated state vector estimate x_k^- and its covariance P_k^- are based on the previous estimate $\hat{x}_{k-1}$ and its covariance $\hat{P}_{k-1}$.

The matrix elements of Kalman gain **K** reflect the weight of created discrepancy in the resultant state estimation. **K** is in a direct ratio to the covariance of the predicted (current) estimate and in an inverse ratio to the covariance of the discrepancy (measurement). The less accurately the predicted estimate (and greater its covariance), the higher the **K** is, and the larger the weight gives to the measurement. Greater gain means rapid response of the measurement filter (the result is close to measurement), small - slow response (the result is close to the predicted estimate). In a private area, this corresponds to a large and small bandwidth of the filter: the narrower the bandwidth, the better the noise suppress and hence the better the filtration quality, the wider - the chances of tracing the target maneuvers are bigger.

3. INTERACTIVE MULTIPLE MODEL (IMM)

It is a Dynamic MM-algorithm. This type of algorithm gives that the nature of the target movement can change at any moment. Therefore, they choose not one model, true for the entire observation interval, but a sequence of pattern changes from the beginning of observation to the current moment including.

Dynamic MM algorithms appear to be optimal for the system, the real state of which changes uneven into a set identical to the set of models used in them. The law change model is usually applied in Markov or semimarkov processes of the first order, with certain transition probabilities π_{is} [5].

The optimal dynamic MM-algorithm for filtering the mixed random process having its N-model composition to form a result-based estimation of states at that point in time must be computed $l = N^k$ estimations to take into account all possible system implementations over time to *k* including , and also evaluate their probabilities. For this, it is necessary to have N^k operating parallel elementary trajectory filters. Types of first and second order algorithms are usually considered, requiring *N* and N^2 filters respectively. In IMM, each filter is triggered with its value, in which it is taken into account the extent to which the model corresponds to the state in which the system is at a time *k-1*.

Let a system of random structure be described with equations:

$$\begin{aligned} x_k &= F(M_{k-1})x_{k-1} + G(M_{k-1})u_{k-1} + w_k \\ z_k &= H(M_{k-1})x_k + v_k \end{aligned} \tag{2}$$

where u_k - an unknown input vector modeling the target maneuver; w_k - Gaussian process noise; v_k - Gaussian noise of observation; system matrices - **F**, **G**, **H**.

The sequence of states $M_k, M_k \in \{M^s\}_{s=1}^{N}$ used in the system is described by the Markov chain with known transitions probabilities: a system located at time point *k-1* in a state M^i switches to a time point *k* in a state M^s with probability π_{is}. The Markov chain is considered stationary, i.e., the transition probabilities do not depend of the current step *k*.

Let Z_k^{k-1} - a set of ticks from the beginning of observation to the time of *k-1*; z_k - ongoing observation. From the previous step we know the probabilities of finding the system in the *i* state $\mu_{k-1}^{i} = p\{M_{k-1}^{i} \mid Z_1^{k-1}\}$, *i = 1, ..., N*, and the estimations $\hat{x}_{k-1}^{i}$ of the filters with covariance P_{k-1}^{i}. The task of an IMM algorithm is to calculate the estimate $\hat{x}_k$ and its covariance P_k at time *k*.

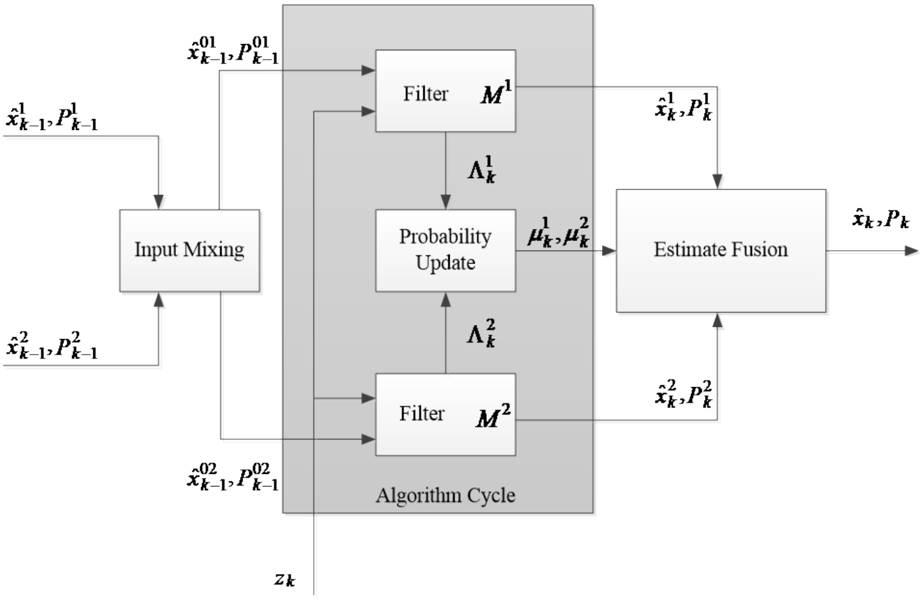

FIG. 2 Flow chart of an IMM algorithm with two models

Fig. 2 shows an IMM structure with two models. An IMM algorithm cycle consists of the following 5 steps [5]:

1) Calculation of the mixing probabilities.
2) Determine the initial conditions of each model filter (mixing estimates).
3) Mode-matched filtering.
4) Mode probability update. Estimation of a posteriori probability for genuineness of the models.
5) Estimate and covariance combination. Calculation of the resultant estimations of the state vector and its covariate matrix.

4. TARGET MOTION MODEL

Constant velocity rectilinear motion

The non-maneuvering motion is a Constant velocity rectilinear motion. The motion equation in this model is the type [5]

$$\mathbf{x}_{k+1} = \mathbf{F}_{CV}\mathbf{x}_k + \mathbf{w}_k \tag{2}$$

where $\mathbf{x} = \left[x\dot{x}y\dot{y}\right]^T$ - state vector; $\mathbf{F}_{CV} = \begin{bmatrix} 1 & \mathrm{T}_0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \mathrm{T}_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ - prediction matrix (T_0 - the time interval between the measurements $\mathbf{x}_k$ and $\mathbf{x}_{k+1}$); $\mathbf{w}_k$ - random sequence in the form of white Gaussian noise with zero mean and a known covariance matrix $\mathbf{Q} = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix}$, σ_x^2 and σ_x^2 - dispersion of noise at the relevant coordinates, which takes into account the impact of random accelerations due to deviation from the course, wind shifting and other factors.

Since there is some deviation from the straight rectilinear movement in this model, it is sometimes called a motion at almost constant velocity, referred to as CV1.

Maneuvering motion with known constant turn.

The model describes a maneuver in which the target moves at a constant velocity ν to turn at a constant turning velocity ω (CT, Coordinated Turn or Constant Turn). A situation where the target is turning at a constant and known turning velocity occurs when civilian aircrafts are tracked to the areas of the aerodrome where the maneuvers of the aircraft are strictly regulated. The state vector and input matrix of the CT model correspond to the CV model, and the predicted matrix has the type [5]

$$\mathbf{F}_{CT} = \begin{bmatrix} 1 & \frac{\sin\Omega T}{\Omega} & 0 & -\frac{1-\cos\Omega T}{\Omega} \\ 0 & \cos\Omega T & 0 & -\sin\Omega T \\ 0 & \frac{1-\cos\Omega T}{\Omega} & 1 & \frac{\sin\Omega T}{\Omega} \\ 0 & \sin\Omega T & 0 & \cos\Omega T \end{bmatrix}.$$

Dynamic Models of flying Aircraft

Let the target location be counted with a sample period T = 1s.

This target moves in a plane initially at a constant rate and speed to the first k = 50 measurements and then performs a coordinated maneuver in the next 50 sample periods at a constant turning velocity. In the third 50 sample periods, it continues his constant course and in the last 50 samples it carried out a second maneuver at the same constant velocity. Using the constant velocity target motion model given in equation (3), the model with the maneuver in equation (4) and the model of measurement in equation (5) simulates the movement of the target (fig.3, 4) [1,4].

$$\mathrm{x}(\mathrm{k}+1) = \begin{bmatrix} 1 & \mathrm{T} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \mathrm{T} \\ 0 & 0 & 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} \frac{1}{2}T^2 & 0 \\ T & 0 \\ 0 & \frac{1}{2}T^2 \\ 0 & T \end{bmatrix} w(k) \tag{3}$$

$$x(k+1) = \begin{bmatrix} 1 & \frac{\sin\Omega T}{\Omega} & 0 & -\frac{1-\cos\Omega T}{\Omega} \\ 0 & \cos\Omega T & 0 & -\sin\Omega T \\ 0 & \frac{1-\cos\Omega T}{\Omega} & 1 & \frac{\sin\Omega T}{\Omega} \\ 0 & \sin\Omega T & 0 & \cos\Omega T \end{bmatrix} x(k) + \begin{bmatrix} \frac{1}{2}T^2 & 0 \\ T & 0 \\ 0 & \frac{1}{2}T^2 \\ 0 & T \end{bmatrix} w(k) \tag{4}$$

$$z(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(k) + v(k) \tag{5}$$

In equations (3) and (4) $w(k)$ is a zero-mean white Gaussian noise with covariance

$\Sigma_w = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$ and $\sigma^2 = 10^{-3}$

In equation (5), $v(k)$ is a zero-mean white Gaussian noise with covariance $\Sigma_v = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}$ with $\sigma_v^2 = 10^{-3}$

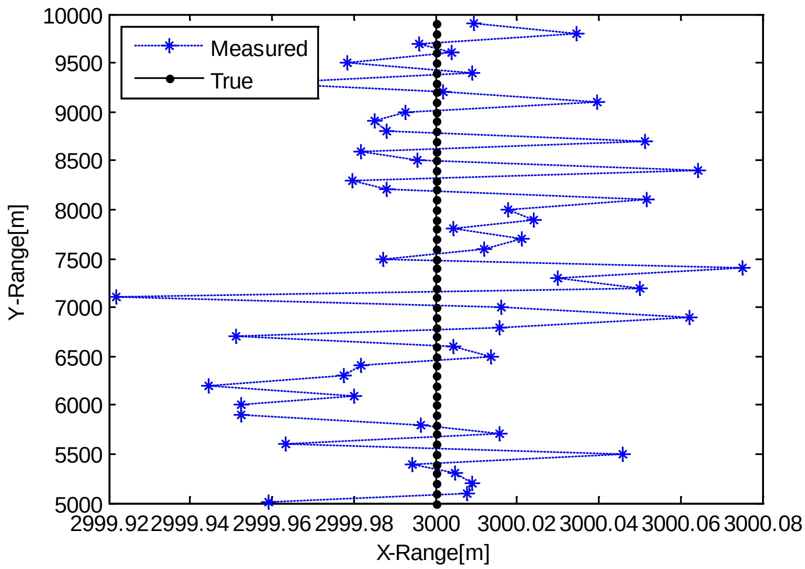

FIG. 3 Target trajectory in the first 50 samples

In Fig. 3 we observe the measured noise of the measurement with blue. Since the target in this section of its trajectory is not shifted to the coordinate x, this noise can be seen how it fluctuates in very small values. In Fig. 4, however, the noise cannot be seen despite its existence, as the entire trajectory of the target is observed, where its x and y coordinate displacements are considerably larger than the noise values.

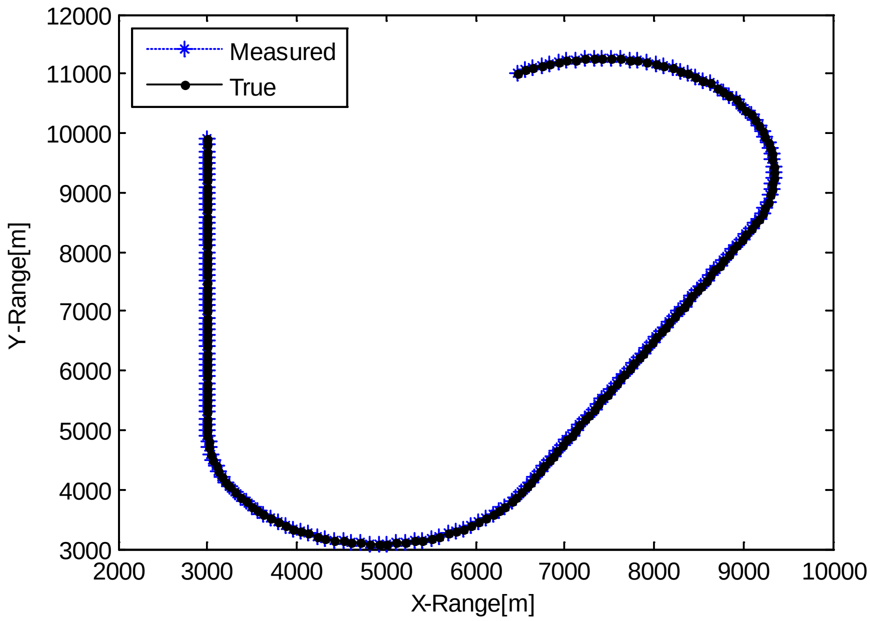

FIG. 4 Target trajectory for the entire 200 sample periods

5. RESULTS OF THE EXPERIMENT.

Kalman filter

Two Kalman filters have been modeled. In one for transition matrix, reflecting the relationship of the previous and last meaning of x, is used $\mathbf{F}_{CV}$, and in the other matrix $\mathbf{F}_{CT}$.

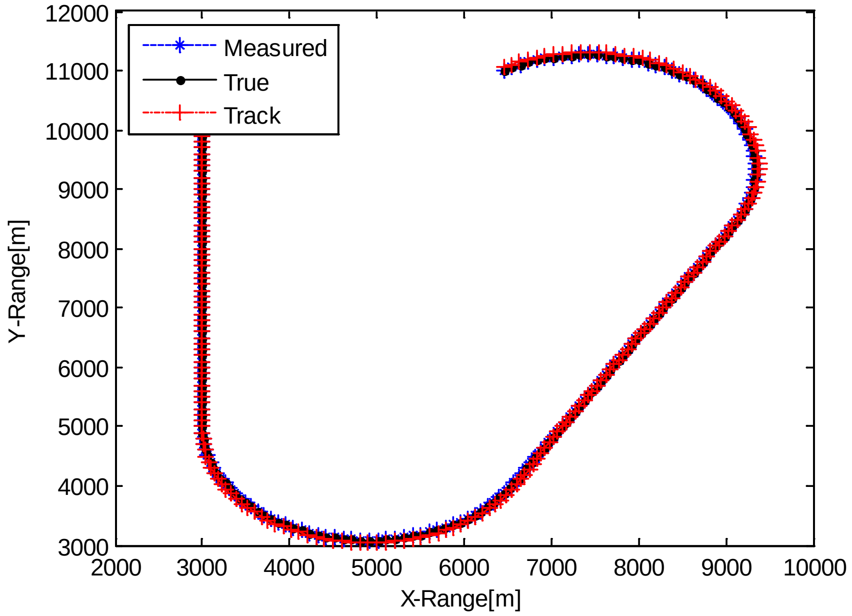

FIG. 5 Tracking with KF in the first model

The first Kalman filter tracks the course at a constant velocity (Fig. 5), the calculated error is insignificant, but when it tracks the maneuver, the calculated error becomes tangible - about 40 meters (Fig. 6).

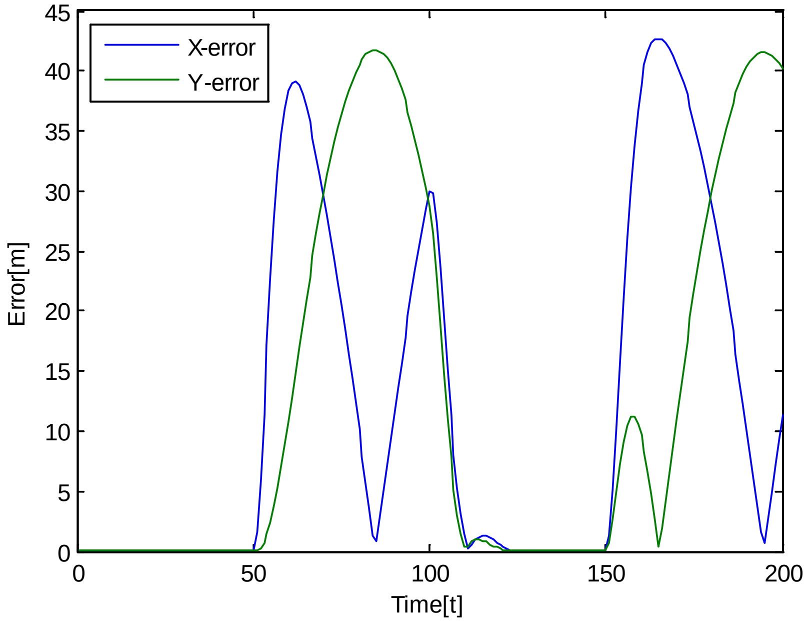

FIG. 6 Estimating of the error in the first model.

On the opposite, the second Kalman filter can accurately track the maneuver (Fig.7), but the estimated error at constant velocity is greater (Fig.8).

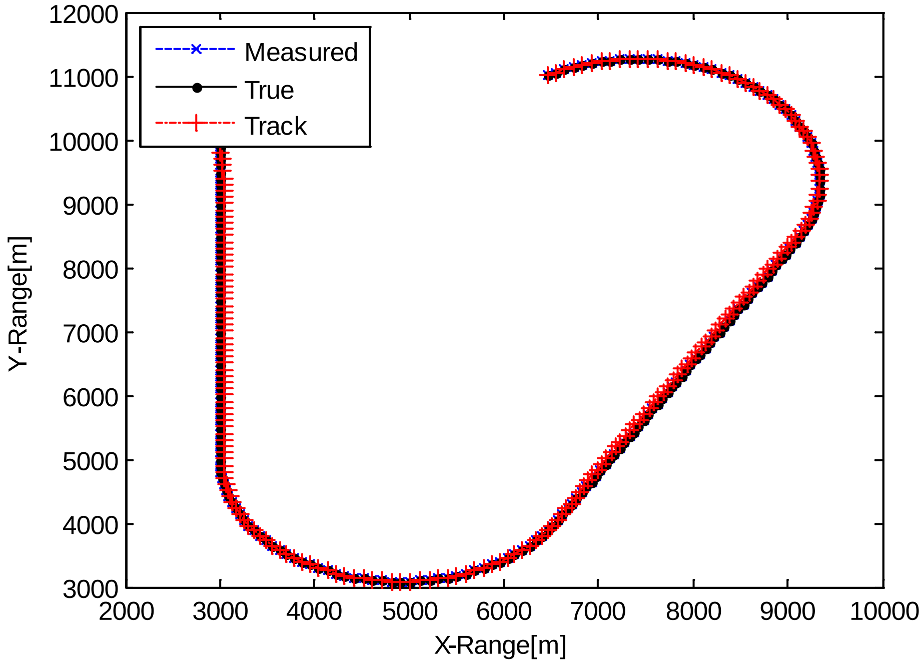

FIG. 7 Tracking with KF in the second model.

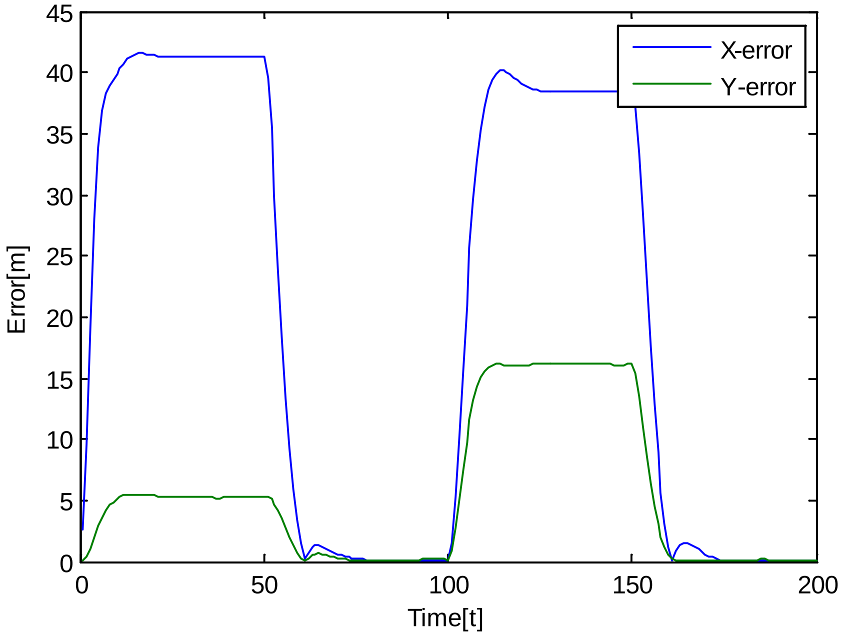

FIG. 8 Estimation of the error in the second model.

Interactive Multiple Model

In this algorithm, there are two models of elemental trajectory filters. The sequence of patterns used in the system is described with a Markov chain with known transition probabilities through the following transition matrix: $[\pi_{ij}] = \begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix}$ [4]. The tracking results are very accurate to the real target movement (Fig.9).

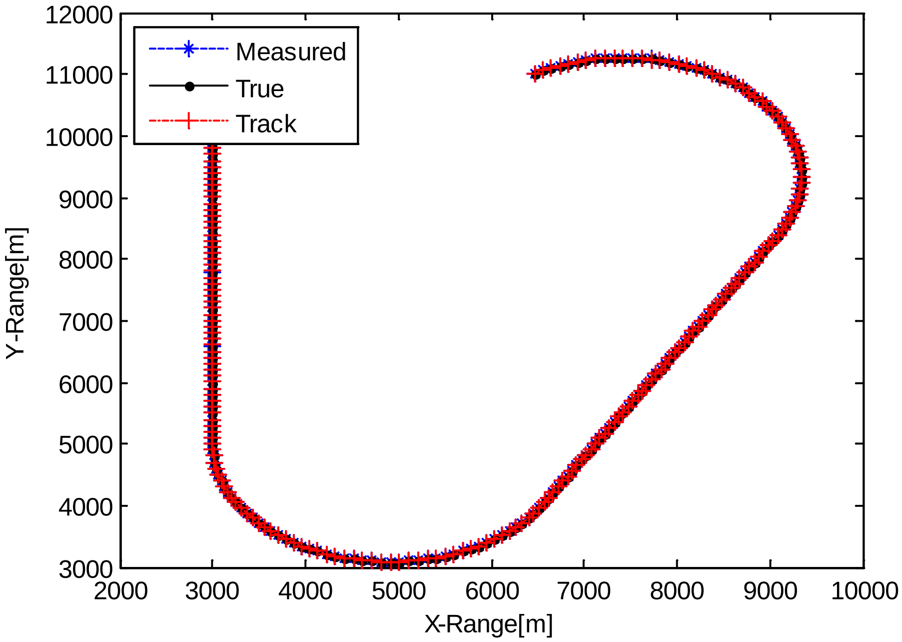

FIG. 9 Tracking with an IMM algorithm

In Fig. 10 shows that the error of the IMM algorithm is less than 0.25 m throughout the entire tracking process.

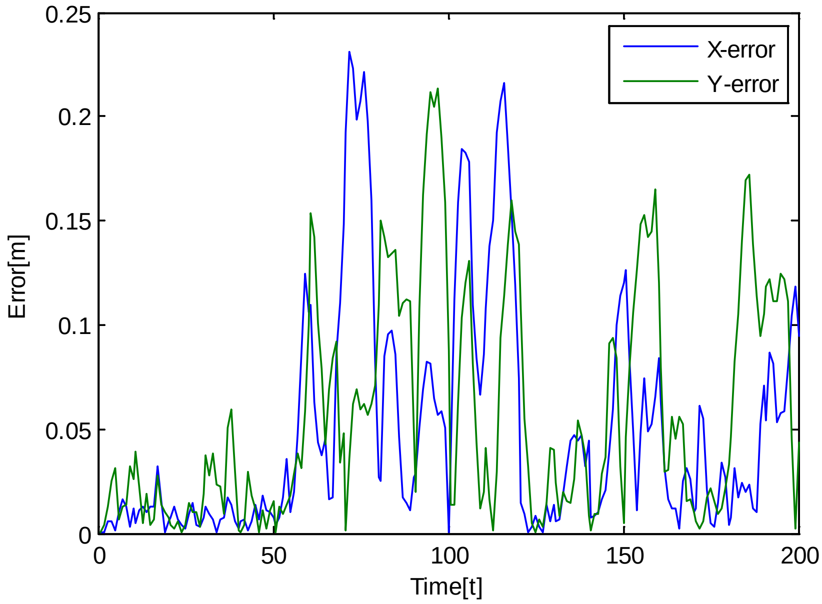

FIG. 10 Estimation of the tracking error with the IMM algorithm.

CONCLUSION

This paper presents two filters through which the target trajectory models are passed. One is a linear Kalman filter, and the other is an Interactive Multiple Model (IMM) algorithm. The results of the experiments show that the performance of the IMM algorithm during target tracking is better than the Kalman filter.

With the simulated input parameters, the Kalman filter tracing errors exceed those of the IMM algorithm up to 40 times. It is precisely the robustness and rapid response to the target maneuver that necessitated the widespread use of the Interactive Multiple Model Estimator.

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