

ASPECTS REGARDING SOME WAYS OF DIFFERENTIATED USE OF TEACHING MEANS IN THE MATHS COURSES

Mirela A. TÂRNOVEANU, Monica A.P. PURCARU

“Transilvania” University of Braşov, Romania
(mi_tarnoveanu@yahoo.com, mpurcaru@unitbv.ro)

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Abstract: *This paper aims at highlighting several aspects regarding the didactic means in terms of their use as a differentiated training strategy for Linear Algebra and Analytical and Differential Geometry courses and seminars. Thus, it is emphasized the importance of the differentiated use of didactic means in the differential treatment of students, some ways of using the didactic means in the courses and seminars are analyzed and exemplified. The paper concludes with the conclusions of the study.*

Keywords: *didactic means, differentiated use*

1. INTRODUCTION

"Inventory of traditional techniques of education and training in Mathematics has been reduced, until recently, to verbal means, chalk, blackboard, pencil and notebook. The palette of these means has expanded considerably, including audio-visual equipment, inclusive the computer, which amplifies the classical possibilities of teaching and learning Mathematics. Today, this process is increasingly under the influence of the technical means of training, designed to increase its efficiency, to facilitate it. It is not possible to conceive today an improvement in mathematical education activities, without ever increasing use of technical means of training. "[1]

"Usually a good professional in the field has also an extensive information complementary to the technical knowledge in the field. Moreover, if the technical aspects are accessible to specialists, the "cultural" ones can receive virtually anyone no matter their specialty. [2]

This "complementary information" is obtained during the faculty, whenever teachers at courses or seminars call for additional bibliography.

"Teachers must select teaching resources so that they can organize attractive and accessible activities, and by their proper choice they can better achieve their goals." [3]

"Differentiation and individualization of education is an old, but always present pedagogical problem, because people differ from each other not only in their way of thinking and being, but also in their capacity and rhythm of learning, their attitude to this one".[4]

Working differentiated with students in Math courses and seminars involves choosing the scientific contents and the training strategies appropriate to their teaching - learning - evaluation according to students possibilities and particularities, and this also means the differentiated use as didactical means.

2. THE IMPORTANCE OF DIFFERENTIATED USE OF DIDACTIC MEANS IN LINEAR ALGEBRA AND ANALYTICAL AND DIFFERENTIAL GEOMETRY TEACHING

"Eliminating the admission exam at most universities has created situations where, at courses they meet from art high school graduates, to laureates of school Olympics. Presenting attractive courses in these conditions is a challenge to scientific research." [5] So, in the current polytechnic higher education, in the vast majority of the groups there are both students with a low level of knowledge at mathematics and students with an average level, or with a high level of knowledge in these disciplines.

Regardless of the level at which the students are, or from which they start, the teacher has to do his job and always find the most useful ways for both the low-level and the high-level to be involved both in the activity at the seminar. Under these conditions, he will increasingly place emphasis on differentiated teaching, this assuming the differentiation of content, but also the differentiated use of methods and didactic means.

Differentiated use of teaching means in math lessons is an important resource in differentiated training. It gives each student the opportunity to learn in his own style. With the help of didactic means used in math courses and seminars, students are able to imagine the abstract theoretical notions, to link their math knowledge to their life experience, to integrate them into "their mathematical dowry," even to enrich them and not just memorize them.

Using non-differentiated means of teaching in the differentiated training of students in mathematics, situations such as those described below can be reached: students who have a high level of knowledge at the discipline announced, the unitary use of the didactic means leads them to boredom, that is for them the activity carried out with the same didactic material is too low, as well as for the students who have a minimal level of knowledge, the unitary use of the didactic means can lead to the further removal of these discipline, ie for them the activity carried out using the same didactic means should be too high and thus do not cope with the requirements. For this reason, during the design of courses and seminars, the teacher has to choose differently, if necessary, the necessary didactic means, so that all the students of the year or of the group can be integrated within them, in order to reduce the level differences between them.

Differentiated use of all mathematical training strategies aims at maximizing the intellectual potential of each student, causing him to overcome his fear, even his own limits sometimes, and to become an active participant in his own training, some of them even get performance.

Didactic means have a greater differentiated use in the courses and seminars of Analytical or Differential Geometry, as it is specific to these discipline to use more didactic means, indispensable in the training for students of the geometric notions.

And last but not least, didactic resources should sometimes be used in a differentiated way so that the teacher can more easily achieve the proposed objectives for all students in the group or the year respectively.

Differentiation of didactic means is an ever-present problem because students differ between themselves both in their own way of thinking and their ability to concentrate, by their attitude towards learning, but also by their own rhythm and style of learning, by hereditary dowry, etc.

3. MODALITIES AND EXAMPLES OF DIFFERENTIATED USE OF DIDACTIC MEANS IN LINEAR ALGEBRA AND ANALYTICAL AND DIFFERENTIAL GEOMETRY TEACHING

Differentiated treatment of students at Linear Algebra and Analytical and Differential Geometry courses or seminars, using also differentiated didactic means, can be achieved both in the teaching and learning process, as well as in their evaluation. Meaning, the didactic means can be differentiated used in these disciplines both in the courses and seminars of acquiring new knowledge, but also in the lessons of skills forming, or in the lessons of recap and systematisation or in those of assessment.

Less often in a course, but more often at seminars, didactic means can be used differentiated in an event of the lesson, or sometimes in two or throughout the activity. In this regard, they can be used differentiated to catching the attention, refresh knowledge, achieve performance, provide feedback, guide the learning, and even for the homework.

It can also be found that the lesson is designed without the didactic means being differentiated, but due to unexpected situations, individual support can only be provided to students who request it, i.e. those who do not meet the requirements of the lesson.

The design of Linear Algebra and Analytical and Differential Geometry courses or seminars, in which didactic means have been chosen for their differentiated use can be done in many ways, all in the idea of obtaining more in-depth feedback that allows the teacher to intervene promptly to help them progress both to the students who have a high level of knowledge in these subjects as well as those with a lower level.

Differentiated use of didactic means is in most cases in close connection with the differentiated use of methods and procedures used in courses or seminars, but also with the differentiation of scientific content. However, there is also the possibility that differentiated use of didactic means can be done without the differentiated use of the methods and procedures used less frequently, even without the differentiation of content.

Here are some examples of differentiated using of didactic means in the Linear Algebra and Analytical and Differential Geometry courses or seminars, that can be selected for their achievement.

In order to make a first classification of the students, the initial evaluation can be done with assessment sheets on two level groups, if there are students in class who have tested mathematics of different level of difficulty at baccalaureate.

A very common way of using differentiated didactic means is the usage of work sheets having different level of difficulty. Thus, in the Linear Algebra seminar, with the theme: "Vector spaces, subspaces, operations with subspaces ", teacher can formulate three work sheets with questions, with varying degrees of difficulty:

For students with a low level in Linear Algebra

1. What algebraic structures learned in high school do you know?
2. What is understood by the commutativity of a composition law?
3. What is understood by the distributivity of scalar multiplication with respect to the addition of vectors?
4. What operations with subspaces do you know?
5. What is the null space?
6. What is meant by the canonical basis of arithmetic space?

For students with a medium level in Linear Algebra

1. What is a vector space? Exemplify.
2. What is meant by supplementary subspaces?
3. What is the subspace spanned by a nonempty subset of a vector space?
4. What is meant by a vector subspace? Exemplify.

5. What is meant by bases and dimension in a vector space? Exemplify.

6. Which is the theorem of the change of basis?

For students with a high level in Linear Algebra

1. What supplementary subspaces do you know?

2. What theorem about the sum of two subspaces you know? Exemplify its use in a category of exercises.

3. What is the necessary and sufficient condition for the sum of the two subspaces to be direct? Exemplify a category of exercises whose sum is calculated with this theorem.

4. What theorem about the subspace dimension do you know and for what purpose you can use it?

A modality of using differentiated didactic means at the course, can be three notes with project' subjects. The subject, taught in high school, for the students who are left behind in these branches of mathematics, can be about the conics on canonical equations and the straight line in plane. The project' subject for middle level students, can be to study a theoretical theme, complementary to the theory taught at the course, with the title : Plane curves often used in the technique: definition, properties (without proofs). The theme of the project for high level students, can be: to study the Titeica and Bertran curves: definition, properties (with proofs).

Individual support can only be provided to students who request it. A such example from the Analytical Geometry seminar on "Conics on general equations" is described below. After a few conics exercises solved on the board, the teacher give the students independent work, to reduce the equation of a conics to the canonical form and then to represent it graphically. All students who did not have in high school math curriculum: study of conics on canonical equations, received a basic information sheet in order to represent graphically the determined canonical equation. Students who graduated from the Mathematics and Computer Science section don't need such a sheet in solving the exercise.

Another example, during the seminars of differential geometry, students with a lower level of knowledge at mathematical analysis receive a sheet with the formulas of derivation of elementary functions and with the primitives of elementary functions, while students with a high level of knowledge, did not need extra didactical means.

Another way of using differentiated didactic means is the election from problem books of problems corresponding students' level of knowledge.

A such example met in several Linear Algebra and Analytical and Differential Geometry seminars on different topics is the followig: in order to achieve feedback through independent work, the teacher conceives two work sheets containing problems with varying degrees of difficulty, namely: for students with a lower level of mathematics knowledge, a sheet with simple problems, and for other students, the worksheet contains problems with medium or high difficulty level.

For example, in the Analytic Geometry seminar, with the theme: "The Euclidian Space of Free Vectors", the two work sheets are:

Sheet A, (Lower Level)

1. The vectors are given:

$$\bar{a}, \bar{b} \text{ și } \bar{c} \text{ cu } \|\bar{a}\| = 1, \|\bar{b}\| = 2, \|\bar{c}\| = 3 \quad \sphericalangle(\bar{a}, \bar{b}) = \frac{\pi}{3}, \sphericalangle(\bar{a}, \bar{c}) = \frac{\pi}{4}, \sphericalangle(\bar{b}, \bar{c}) = \frac{\pi}{6}.$$

Calculate the vector norm $\bar{a} + \bar{b} - \bar{c}$.

2. Let: $\vec{r}_A = 2\vec{i} + \vec{j} + \vec{k}$, $\vec{r}_B = -5\vec{k}$, $\vec{r}_C = \vec{i} + 3\vec{j} - \vec{k}$, $\vec{r}_D = \vec{i} - \vec{j} + 2\vec{k}$, be the position vectors of points A, B, C and D. Determine: the scalar product of the vectors \overline{AB} and

\overline{AC} , the vector product of the vectors \overline{AB} and \overline{AC} and the mixed product of the vectors \overline{AB} , \overline{AC} , \overline{AD} .

Sheet B, (Higher Level)

1. The vectors are given:

$$\overline{a}, \overline{b} \text{ și } \overline{c} \text{ cu } \|\overline{a}\| = 1, \|\overline{b}\| = 2, \|\overline{c}\| = 3, \sphericalangle(\overline{a}, \overline{b}) = \frac{\pi}{3}, \quad \sphericalangle(\overline{a}, \overline{c}) = \frac{\pi}{4},$$

$\sphericalangle(\overline{b}, \overline{c}) = \frac{\pi}{2}$. Calculate the parallelepiped volume built on the representatives by oriented

segments to the common origin of the three free vectors.

2. Points A (1, -5, 4), B (0, -3, 1), C (-2, -4, 3), D (4, 4, -2) are the vertices of a tetrahedron. Calculate the height of the tetrahedron from vertex A.

Other modalities of using differentiated means of teaching at Linear Algebra and Analytical and Differential Geometry seminars appears when using variants of interactive methods, such as "Gallery tour", "Jigsaw", "RAI", "Cube", etc., as differentiated training strategies, the teacher has to draw up differentiated work sheets or differentiated theoretical subtopics, or even two sets of materials with varying degrees of difficulty.

Such an example is found in the case of the interactive method: "Jigsaw", used at the seminar on the theme: "Eigenvalues, eigenvectors", where after solving various problems in this unit of learning, in order to obtain the feedback, the group can be divided into three homogeneous subgroups, depending on the knowledge of the students in the Linear Algebra, each subgroup having the task to solve a problem-a subtopic from the expert worksheet, corresponding to his level of knowledge.

The expert worksheet for the seminar: "Eigenvalues, eigenvectors" is given below.

Subtopics of expert worksheet:

1. Exercise to determine the eigenvalues of a matrix A or a linear transformation T. (Low level)
2. Exercise to determine the inverse matrix and the value of a matrix polynomial using the Hamilton-Cayley theorem. (Medium level)
3. Exercise to determine the diagonal form a matrix and the basis corresponding to the diagonal form. (High level)

Another example of using differentiated didactic means at the Analytic Geometry seminar on "Plane and straight line in space," is the following: students are asked to demonstrate using a drawing on an A3 sheet and colored pencils, the formula: parallelepiped volume = 6x the tetrahedron volume as follows: lower-level students demonstrated this formula on the particular case of the rectangular parallelepiped, and students with a higher level of knowledge, demonstrated this formula on the general case of parallelepiped.

Another example from the Analytic Geometry course about "Quadrics", for students to be motivated to learn and to see the applicability of the taught subject, thus linking the situations encountered in life and what was taught in the course it was completed by presenting with video projector numerous buildings, bridges, railways, in which some of the quadrics were used. In order to achieve a continuous and differentiated assessment of the students' knowledge of quadrics, each student was asked to bring such examples in the next hour of the seminar to identify the quadrics learned.

Each student have to identify two of the quadrics learned in the pictures and presented some of their properties as follows: for lower- level students the quadrics are: the sphere and the ellipsoid, for middle- level students the quadrics are: the cone and cylinders, and for higher level students, the quadrics are: the hyperboloid of one sheet and the hyperbolic paraboloid.

At some of the Linear Algebra and Analytical and Differential Geometry seminars, the homework can be given using two work sheets with problem having two level of difficulty, thus, students will be given a set of problems to solve, whose degree of difficulty corresponds to their level of preparation for algebra or geometry.

4. CONCLUSIONS

Upon entering college, there are great differences between math students 'knowledge, so that the differentiated use of didactic means is absolutely necessary, contributing to students' progress in these disciplines. The success of a course or a seminar depends also on how the teachers use the didactic means during their lesson.

It is natural that when the teaching of mathematical notions is done using differentiated scientific content, or differentiated procedural resources and didactic means to be used differentiated too.

Using didactic means at both courses and seminars of Linear Algebra and Analytical and Differential Geometry, as a differentiated training strategy, can be done in any kind of lesson and at any event of it.

If in the Linear Algebra and Analytical and Differential Geometry seminars or courses the didactic means are also used differentiated, then the chances of achieving faster the objectives proposed for the lesson increase, thus improving the activity.

There are certainly many other variants of using differentiated didactic means, the choice of one or other of them depends both on the situation in the classroom, the theme of the seminar or course, and the talent of the teacher.

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