REGARDING A CONTINUOUSLY DIFFERENTIABLE FRICTION MODEL USED FOR CONTROL OF DYNAMIC SYSTEMS DESIGN

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Abstract: *This paper analyzes several models that have been useful to characterize friction in motion control systems, where for high-performance of different engineering systems, model-based controllers are required to accommodate for the system nonlinearities. Unfortunately, according to the dynamical systems theory, developing accurate models for friction has been historically challenging, because typical models were either discontinuous or only piecewise continuous. Motivated by the fact that discontinuous and piecewise continuous friction models are problematic for the development of high-performance continuous controllers, a new model for friction is considered and analysed in this paper. This simple continuously differentiable friction model represents a foundation that captures the major effects existed in the friction modeling area of applied mathematics and physics.*

Keywords: *motion control systems, friction, dynamic models, optimal control systems.*

1. INTRODUCTION

The dynamical systems theory [1] provide us many dynamical systems (physical, chemical, economical, ecological etc) [2] that can be modeled by different mathematical relations. In this context, we relate to stochastic differential and/or difference equations [3]. As their name suggest, these mathematical systems change with time or any other independent parameters according to the dynamical relations.

In this case, a mathematical model (as presented in **Fig. 1a**) is a precise representation of a system's dynamics used to answer questions via numerical analysis and computers simulation, describing, at the same time, the input/output behavior of systems [13].

The input given to the system corresponding to this best situation is called "optimal" control [4]. For high-performance engineering systems, model-based controllers are typically required to accommodate for the system nonlinearities [14].

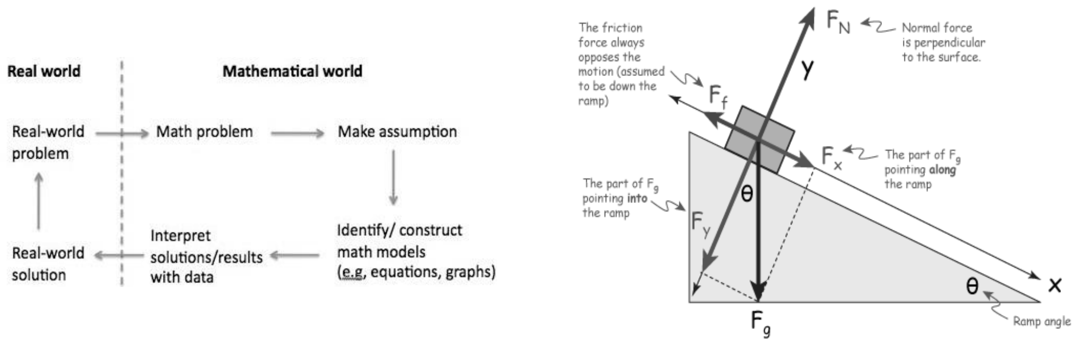


FIG. 1. a) Mathematical models as a bridge between real and mathematical world; b) The description of the friction behavior (source: math4teaching.com/mathematical-modeling / www.drcruzan.com/friction.html)

Unfortunately, developing accurate models for friction is difficult. Typical models are either discontinuous and many other models are only piecewise continuous [12]. Friction is generally described as the resistance to motion [9], when two surfaces slide against each other. On the other hand friction can also cause undesirable effects. For high precision mechanical motion systems for example, friction can deteriorate the performance of the system, so the possible unwanted consequences caused by friction appear to be: *tracking errors, limit cycling and hunting* [7]. In motion control a possible way to minimize the influences of friction is to compensate for it (**Fig. 1b**).

2. FRICTION MODELS: PROPERTIES AND CLASSIFICATION

Motivated by the desire to develop an accurate representation of friction in systems, various control researchers have developed different analytical models. In general, the dominant friction components that have been modeled include: *static friction* (the torque that opposes the motion at zero velocity), *Coulomb friction* (the constant motion opposing torque at non-zero velocity), *viscous friction* (when full fluid lubrication exists between the contact surfaces), *asymmetries* (different friction behavior for different directions of motion), *Stribeck effect* (at very low speed, when partial fluid lubrication exists, contact between the surfaces decreases and thus friction decreases exponentially from stiction), *position dependence* (oscillatory behavior of the friction torque due to small imperfections on the motor shaft and reductor centres, as well as ball bearings elastic deformation.

For situations where the starting friction is higher than friction at a nonzero velocity "static" friction force (F_s) can be distinguished as in **Fig. 2a-c**. For the most common situations the friction decreases with increasing velocity for a certain velocity regime. This is called the Stribeck effect and is shown in **Fig. 2d**.

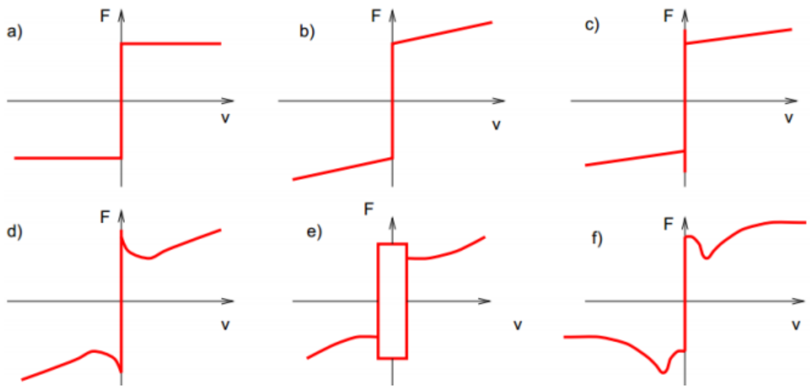


FIG. 2. The representation of the basic static friction force effects [9]: a) Coulomb; b) Coulomb + viscous; c) Coulomb + viscous + stition; d) Stribeck effect; e) Karnopp; f) Hess and Soom Armstrong.

These basic models describe a static relationship between the friction force and velocity. At rest where the velocity is zero, the friction force cannot be described as a function of velocity alone. For practical high accurate positioning systems however other frictional properties have to be considered for a satisfying model.

The models described so far are all empirical-based models. An other branch of friction models is the so called *physics-motivated friction models*. These describe friction on three different physical levels namely on *atomic-molecular* [6], *asperity-scale* [10] and at *tectonic-plate level* [15,16].

Although the physics-based models are capable of capturing all friction-induced phenomena that are observed so far, they are much too complicated for online control purposes, or for the optimal control.

3. THE ANALISYS OF THE DYNAMIC SYSTEMS DESIGN USING A CONTINUOUSLY DIFFERENTIABLE FRICTION MODEL

The scientific literature that we consulted provide us a few options available for characterizing the friction models, such as:

- *adaptive compensation* with differential filter & fuzzy controller (full-states & position measurements);
- characterization of the limit cycles caused by friction (*stick-slip*, *"jump" motion*);
- identification and stability analysis of friction using *Lyapunov's method*;
- modeling and simulation of the dynamic behavior of systems with friction, using *continuosly differentiable model* (6 terms parameterizable form with tanh).
- optimal control and friction estimation using LQR (linear quadratic regulator), PID (proportional-integralderivative), SVM (support vector machine) and AAD (adaptive anti-disturbance);

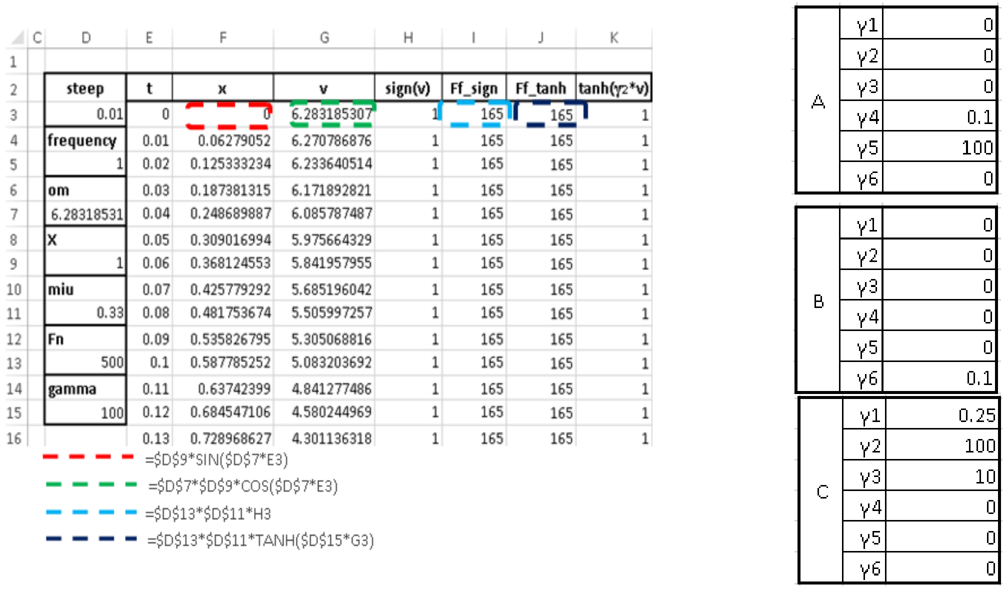
Starting from the last one and according to the general Euler-Lagrange dynamic system, which is following the nonlinear dynamic model characteristics, we can considered - in accordance with [12] - that the friction behavior could be model by a friction term *f(q')* that is assumed to have the following non-linear parameterizable form:

$$f(q')=\gamma_1(\tanh(\gamma_2 q')-\tanh(\gamma_3 q'))+\gamma_4(\tanh(\gamma_5 q')+\gamma_6 q' \quad (1)$$

where γ_i denote unknown positive constants.

The above friction model (1) has the following properties (as resulted from **Fig. 3**):

- is symmetric about the origin and the static coefficient of friction can be approximated by $\gamma_1+\gamma_4$;
- the term $\tanh(\gamma_2 q')-\tanh(\gamma_3 q')$ captures the Stribeck effect where the friction coefficient decreases from the static coefficient of friction with increasing slip velocity near the origin;
- the Coulombic friction coefficient is present in the absence of viscous dissipation and is modelled by the term $\gamma_4(\tanh(\gamma_5 q')$, and a viscous dissipation term is given by $\gamma_6 q'$.



	C	D	E	F	G	H	I	J	K
1									
2		steep	t	x	v	sign(v)	Ff_sign	Ff_tanh	tanh(γ2*v)
3		0.01	0	0	6.283185307	1	165	165	1
4		frequency	0.01	0.06279052	6.270786876	1	165	165	1
5		1	0.02	0.125333234	6.233640514	1	165	165	1
6		om	0.03	0.187381315	6.171892821	1	165	165	1
7		6.28318531	0.04	0.248689887	6.085787487	1	165	165	1
8		X	0.05	0.309016994	5.975664329	1	165	165	1
9		1	0.06	0.368124553	5.841957955	1	165	165	1
10		miu	0.07	0.425779292	5.685196042	1	165	165	1
11		0.33	0.08	0.481753674	5.505997257	1	165	165	1
12		Fn	0.09	0.535826795	5.305068816	1	165	165	1
13		500	0.1	0.587785252	5.083203692	1	165	165	1
14		gamma	0.11	0.63742399	4.841277486	1	165	165	1
15		100	0.12	0.684547106	4.580244969	1	165	165	1
16			0.13	0.728968627	4.301136318	1	165	165	1

A	γ1	0
	γ2	0
	γ3	0
	γ4	0.1
	γ5	100
	γ6	0
B	γ1	0
	γ2	0
	γ3	0
	γ4	0
	γ5	0
	γ6	0.1
C	γ1	0.25
	γ2	100
	γ3	10
	γ4	0
	γ5	0
	γ6	0

FIG. 3. The representation of the non-linear parameterizable friction model construction

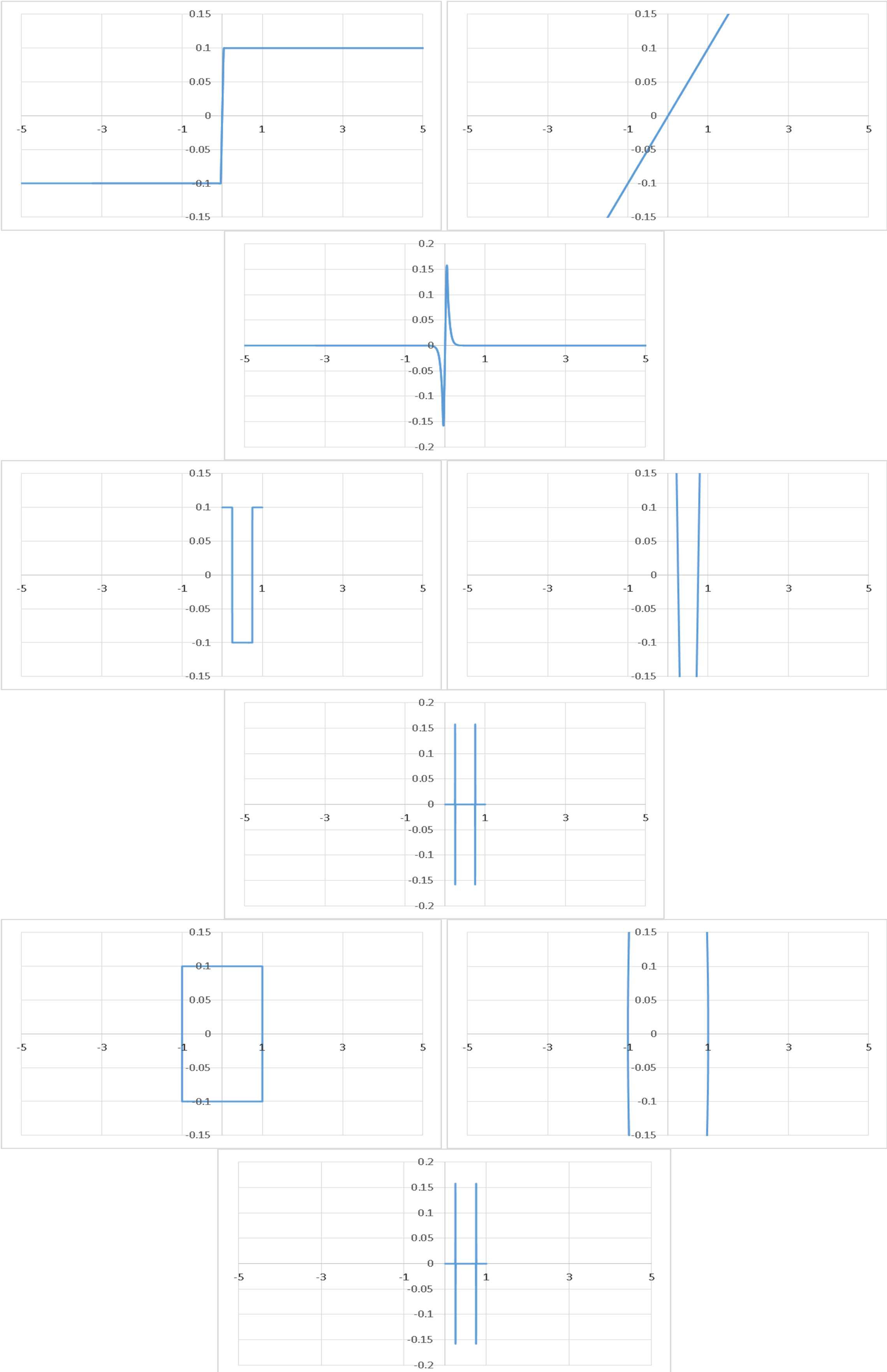


FIG. 4. The representation of **F_f(A,B,C) vs. speed (v)** / **time (t) / displacement (x) -** case no. 1: **steep (0.001)**, frequency (1), displacement – x (1)

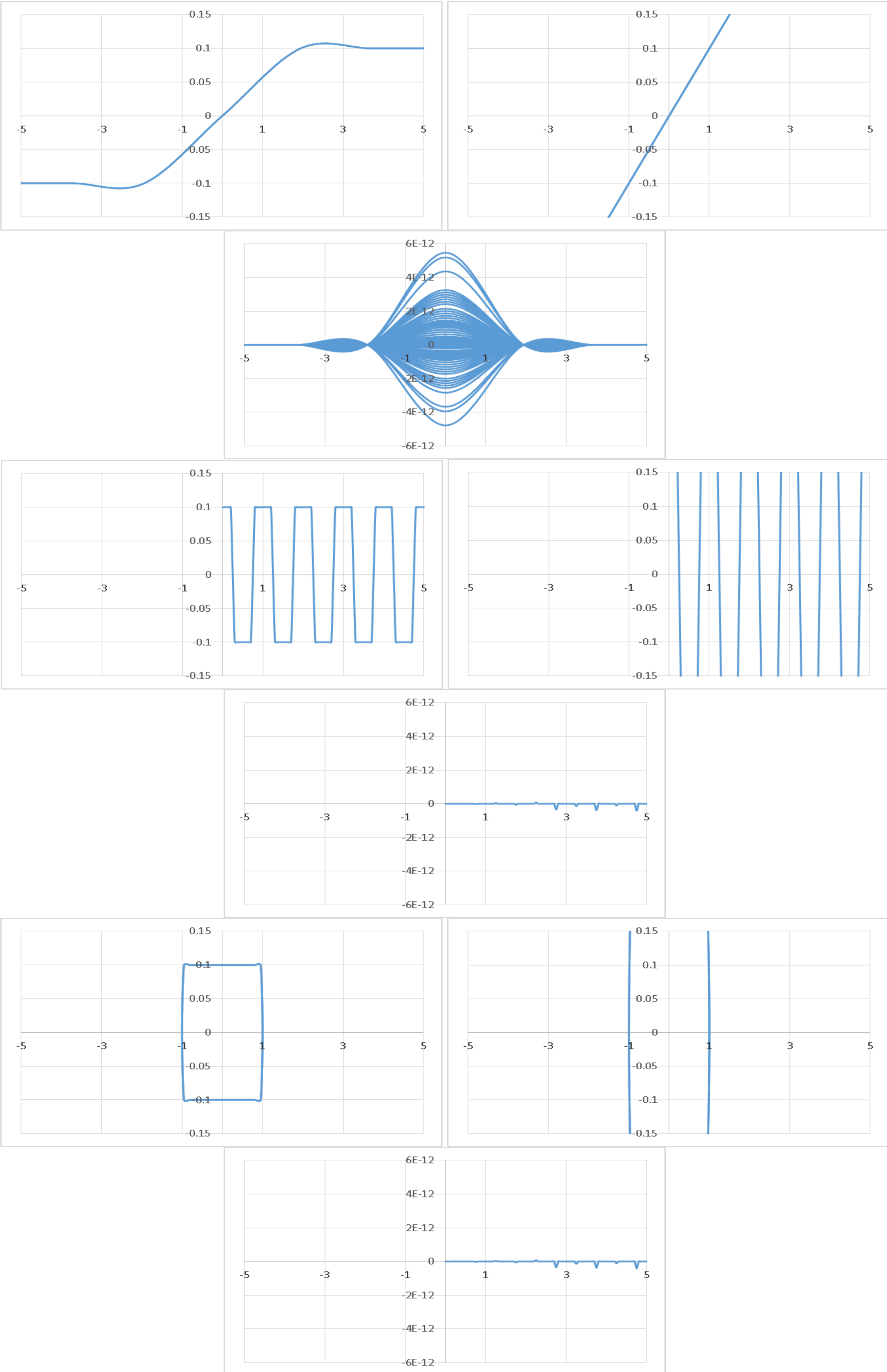


FIG. 5. The representation of **F_f(A,B,C) vs. speed (v)** / **time (t) / displacement (x) -** case no. 2: **steep (0.05)**, frequency (1), displacement – x (1)

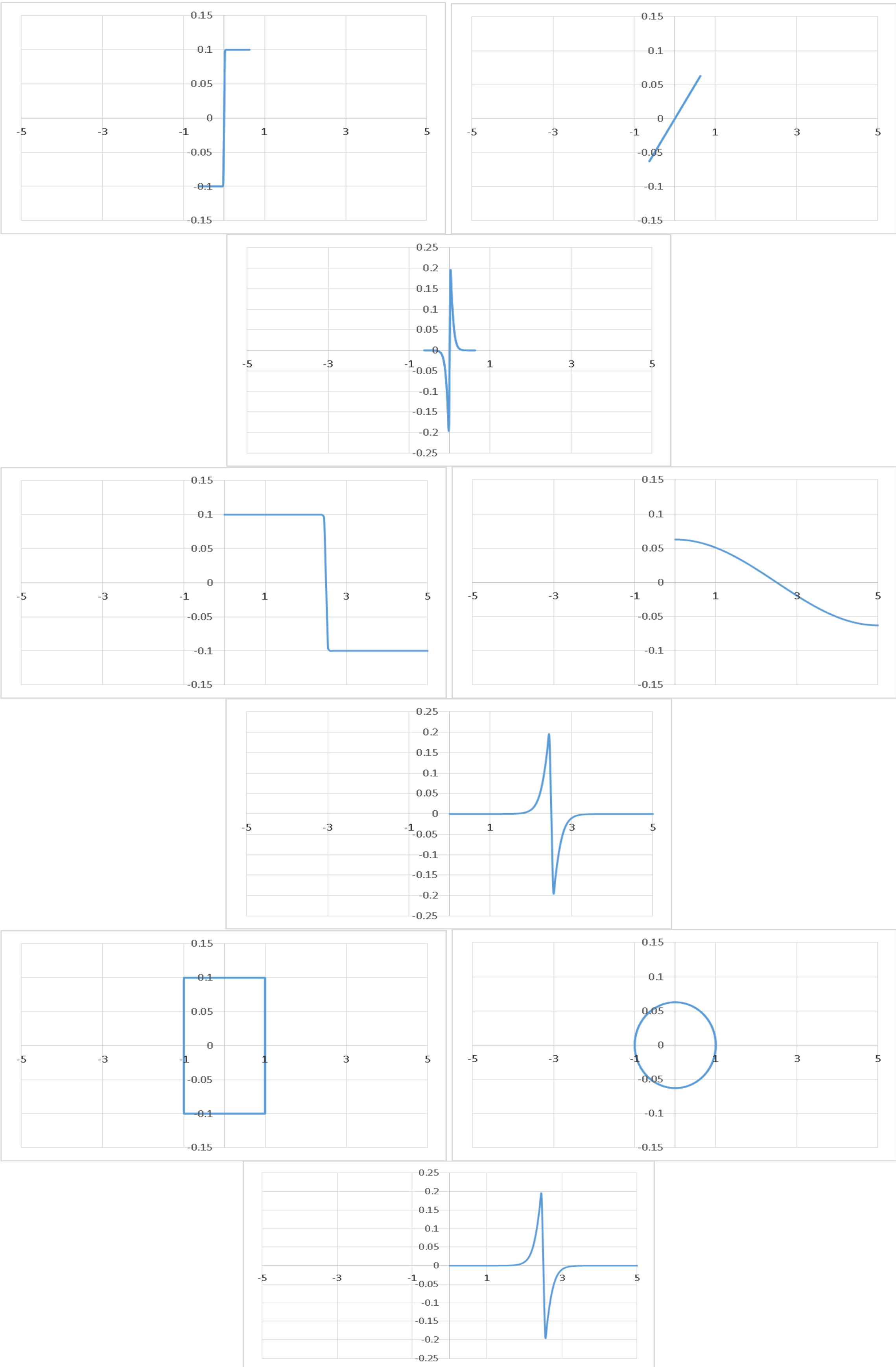


FIG. 6. The representation of **F_f(A,B,C) vs. speed (v) / time (t) / displacement (x) -** case no. 2*: **steep (0.05)**, **frequency (0.1)**, displacement – x (1)

CONCLUSIONS

The friction phenomenon is not considered in system modeling and controller design; however, it has many damaging effects on operation of systems and controllers. Hence, a friction compensation is needed. It is possible to steer these systems from one state to another state by the application of some type of external inputs or controls. If there are different ways of doing the same task, then there may be one way of doing it in the "best" way. This best way can be minimum time to go from one state to another state, or maximum thrust developed by a rocket engine.

The input given to the system corresponding to this best situation is called "optimal" control. For high-performance engineering systems, model-based controllers are typically required to accommodate for the system nonlinearities.

In the present paper we focus on the representation of the non-linear parameterizable friction model construction, so we present the friction force F_f(A,B,C) vs. speed (v) / time (t) / displacement (x) and 3 cases as follow:

- case no. 1: steep (0.001), frequency (1), displacement – x (1);
- case no. 2: steep (0.05), frequency (1), displacement – x (1);
- case no. 2*: steep (0.05), frequency (0.1), displacement – x (1).

with some notable results under computational representation.

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