DYNAMICS OF RECOIL SIMULATION DEVICE POWERED BY CARBON DIOXIDE

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Abstract: *The paper is focused on the dynamics of the recoil simulation device powered by the carbon dioxide (CO_2). The objective is to formulate a mathematical model simulating the cylinder pressurization, and the displacement of the recoil simulation device moving part. The CO_2 is considered as the real gas. The speed of sound in a two-phase saturation is taken into account for the determination of the critical gas flow rate from the CO_2 pressure tank into the cylinder. The problem is solved using the MATLAB environment and results of the theoretical solution are verified experimentally.*

Keywords: *recoil, recoil simulation, gas gun, carbon dioxide, discharge, speed of sound*

1. INTRODUCTION

Employing virtual shooting range has become a solution to increase the safety and effectiveness of shooting and tactical training, at the same time to reduce the cost of training, and the toxic wastes produced by shooting with blank or live ammunition. Recoil simulation becomes an integral part of simulated training weapons used in the virtual shooting ranges.

The recoil simulation devices use the energy of compressed gas from an external source instead of the traditional powder charge to propel the moving part. A positive advantage gained from using CO_2 as a propellant is that a small volume of liquid can convert to a large volume of pressurized gas, thus, the carbon dioxide tanks tend to be smaller and lighter compared to high-pressure air tanks while yielding the same or more shots per fill. Carbon dioxide was the first propellant used in paintball and airsoft technology and due to its low cost, the carbon dioxide has been being widely utilized as a power gas for gas guns and for a recoil simulation.

There is a number of commercial recoil simulation devices on the market, for example [1-3], but there is almost no available literature on dynamics of the recoil simulation device. There are several related publications have been appeared in the last few years documenting the study of phase behavior of carbon dioxide as the power gas for gas guns [4], the simulation of the sound effect of RPG-7 anti-tank grenade launcher for shooting training [5], and the possibility to develop an advanced non-equilibrium model of depressurization in CO_2 two-phase fluids [6].

In this paper, a novel mathematical model simulating the recoil simulation device dynamics is formulated. In order to verify the model, an experimental apparatus is also established.

2. PRINCIPLE OF CO_2 POWERED RECOIL SIMULATION DEVICE OPERATION

Figure 1 shows the basic concept of a CO_2 powered recoil simulation device consisting of a piston, a control valve system, and a return spring. The cylinder is connected with the pressure tank through the control valve system that controls the amount of CO_2 discharged from the pressure tank into the cylinder.

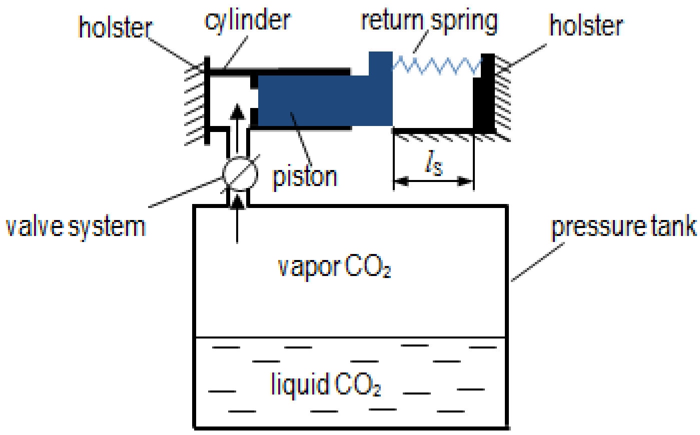

FIG. 1. Schematic of the recoil simulation device powered by CO_2

When the valve is opened by the trigger, a certain amount of CO_2 vapor discharges out of the pressure tank through the valve into the cylinder, in which the pressure increases rapidly, acts on the front of the piston and causes the piston to move backward. At the moment, when the piston reaches its working stroke, it reaches the impact velocity and collides with the device holster on the back position resulting in generating the impact force. After that, the piston returns to the initial position by the return spring force. Then, it collides with the gun holster on the front position and generates the impact force. The resultant force of impact forces and the return spring force is transmitted from the device to the mount and causes the recoil effect.

3. MATHEMATICAL MODEL

3.1. Description and bases of the mathematical model

The above schematic of the CO_2 powered recoil simulation device can be replaced by the thermodynamic system shown in Fig. 2.

Depending upon the initial temperature and pressure within the pressure tank, the CO_2 can exist in the form of single- or two-phase fluid. In the case of single phase CO_2 can exist in the form of vapor if it is superheated, or in liquid form, if the pressure tank is supercharged. In this work, we assume that in the pressure tank CO_2 exists in a liquid-vapor equilibrium, in which the vapor phase occupies the upper part of the tank and the bottom tank portion is occupied by the liquid phase. There exists an interface between the two phases. The change in volume dV/dt is caused by the displacement of the moving part, i.e. the piston.

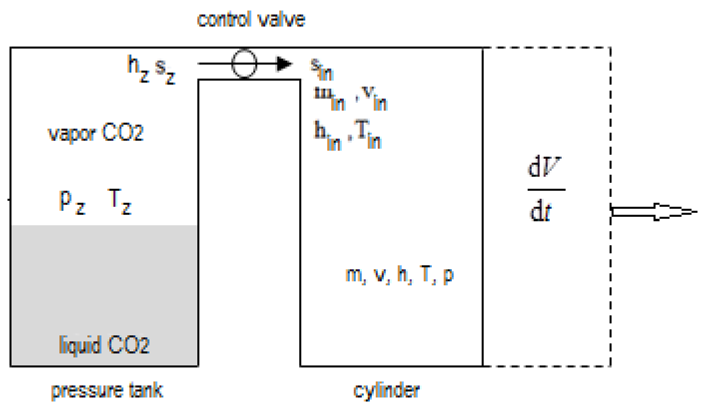

FIG. 2. Open thermodynamic system

3.2. Assumptions

The mathematical model is developed on the basis of the following assumptions:

- Open thermodynamic system.
- Carbon dioxide is considered as a real gas.
- The one-dimensional flow of heterogeneous fluid.
- Two-phase or single phase fluid in the thermal equilibrium.
- In two-phase thermal equilibrium: specific enthalpy, specific volume, density and specific capacities of each phase are functions of temperature or pressure. In the case of single-phase: specific enthalpy and specific capacities are functions of temperature and pressure.
- The isentropic expanding flow of CO_2 through the discharge orifice of the control valve.
- No heat transfer between the fluid within the tank and its walls.
- Control valve opening/closing is immediate, i.e. without any time delay.
- No gas leakage.

The study of the phase behavior of thermodynamic quantities (i.e. pressure, temperature, mass, etc.) within the pressure tank is not the goal of this paper. The comprehensive study of the thermodynamic quantities can be found in [4]. In this paper, we consider the constant values of the thermodynamic quantities inside the pressure tank.

3.3. Equations of the mathematical model

The exchange of mass and energy within the cylinder takes place in the open thermodynamic system. During the unsteady-state processes, when the state quantities of a system vary in time and also the state and the amount of incoming and outgoing fluid can vary in time, the first law of thermodynamics acquires the common differential rate form with respect to a time-lag dt [7]

$$\sum \frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\mathrm{d}U}{\mathrm{d}t} + p\frac{\mathrm{d}V}{\mathrm{d}t}. \tag{1}$$

The change in enthalpy appearing in Eq. (1) is given by the specific enthalpy, h_{in}, and the mass flow rate, $\dot{m}_{\mathrm{in}}$, that are incoming from the pressure tank into the cylinder

$$\sum \frac{\mathrm{d}H}{\mathrm{d}t} = \dot{m}_{\mathrm{in}} h_{\mathrm{in}}. \tag{2}$$

The change in the internal energy within the cylinder appearing in Eq. (1) is given by

$$\frac{\mathrm{d}U}{\mathrm{d}t}=\frac{\mathrm{d}(um)}{\mathrm{d}t}=m\frac{\mathrm{d}u}{\mathrm{d}t}+u\frac{\mathrm{d}m}{\mathrm{d}t}, \tag{3}$$

where m and u account for the mass of CO_2 and the specific internal energy within the cylinder, respectively.

There is the only incoming CO_2 mass flow rate $\dot{m}_{\mathrm{in}}$. Thus, the change in mass of CO_2 inside the cylinder is given by

$$\frac{\mathrm{d}m}{\mathrm{d}t}=\dot{m}_{\mathrm{in}}, \tag{4}$$

The change in the specific internal energy in the above equation can be determined using the following formula [7]:

$$\frac{\mathrm{d}u}{\mathrm{d}t}=c_{\mathrm{v}}\frac{\mathrm{d}T}{\mathrm{d}t}-\left[T\left(\frac{\partial p}{\partial T}\right)_v-p\right]\frac{\mathrm{d}v}{\mathrm{d}t}\cdot \tag{5}$$

The specific heat capacity at constant volume c_{V} for gaseous CO_2 appearing in the above equation is given by the difference of the specific heat capacity at constant pressure c_{p} and the specific gas constant r. The values of c_{p} are available in [8].

The thermodynamic quantities in the output of the valve depend on the pressure p in the cylinder. We consider the phenomena occurring at the beginning of the process, as the first amount of gaseous phase flows into the cylinder. Assuming that the initial pressure p equals to the pressure of the surrounding environment, thus, the mixture appearing at the output of the valve includes dry ice in the form of small solid particles and vapor of CO_2. At the valve output, the mixture temperature T^* is a function of the cylinder pressure p and it is interpolated from the saturation curves. After entering to the cylinder, this two-phase fluid expands to whole cylinder volume, where its equilibrium state is broken due to the change in temperature T within the cylinder. The initial temperature T is assumed to be the surrounding temperature. It causes that the most of the dry ice particles sublimate. Hence, the mixture in the cylinder is in metastable state until the temperature and pressure conditions of the two-phase mixture are met. Therefore, the presence of residual dry ice particles within the cylinder can be neglected. Then, it is possible to describe the thermodynamic state of the gas inside the cylinder by using the van der Waals equation of state [7] that can be written in form

$$p=\frac{rT}{v-b}-\frac{a}{v^2}, \tag{6}$$

where r is the CO_2 specific gas constant, constant a provides a correction for the intermolecular forces and the constant b represents a correction for finite molecular size. The values of constants a, b may be obtained by the theorem of corresponding states [7] using the CO_2 critical point conditions.

From Eq. (6), we obtain the partial derivative appearing in Eq. (5) as

$$\left(\frac{\partial p}{\partial T}\right)_v=\frac{r}{v-b}\cdot \tag{7}$$

The time change in the specific volume appearing in Eq. (5) is given as

$$\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{1}{m}\frac{\mathrm{d}V}{\mathrm{d}t}-\frac{V}{m^2}\frac{\mathrm{d}m}{\mathrm{d}t}, \tag{8}$$

where V is the total volume of the cylinder and its change in time is given by the chamber cross-sectional area A_{p} and the velocity v_{p} of the moving part as bellow

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -v_\mathrm{p} A_\mathrm{p} \tag{9}$$

Introducing Eq. (7) and Eq. (8) into Eq. (5) yields

$$\frac{\mathrm{d}u}{\mathrm{d}t} = c_\mathrm{v}\frac{\mathrm{d}T}{\mathrm{d}t} - \frac{1}{m}\left(\frac{a}{v^2}\frac{\mathrm{d}V}{\mathrm{d}t} + \frac{a}{v}\frac{\mathrm{d}m}{\mathrm{d}t}\right). \tag{10}$$

The internal energy of the cylinder is given by

$$u = h - pv. \tag{11}$$

The specific enthalpy h is a function of pressure p and temperature T within the cylinder:

$$h = h(p,\, T). \tag{12}$$

By introducing Eq. (4), Eq.(10) and Eq. (11) into Eq. (3), then introducing Eq. (2) and Eq. (3) into Eq. (1), and by rearranging we obtain the time change in temperature within the cylinder in form

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{1}{mc_\mathrm{v}}\left[\dot{m}_\mathrm{in}\left(h_\mathrm{in} - h - \frac{a}{v} + pv\right) + \left(\frac{a}{v^2} - p\right)A_\mathrm{p} v_\mathrm{p}\right]. \tag{13}$$

At the control valve output, carbon dioxide exists in the two-phase mixture in solid – vapor or liquid – vapor equilibrium. The specific volume v_in, specific enthalpy h_in, specific entropy s_in and the density ρ_in of the two-phase mixture at the valve output are determined as

$$v_\mathrm{in} = \gamma v''_\mathrm{in} + (1-\gamma)v'_\mathrm{in}, \tag{14}$$

$$h_\mathrm{in} = \gamma h''_\mathrm{in} + (1-\gamma)h'_\mathrm{in}, \tag{15}$$

$$s_\mathrm{in} = \gamma s''_\mathrm{in} + (1-\gamma)s'_\mathrm{in}, \tag{16}$$

$$\rho_\mathrm{in} = \gamma \rho''_\mathrm{in} + (1-\gamma)\rho'_\mathrm{in}, \tag{17}$$

where $v'_\mathrm{in} = v'_\mathrm{in}(p)$, $v''_\mathrm{in} = v''_\mathrm{in}(p)$, $h'_\mathrm{in} = h'_\mathrm{in}(p)$, $h''_\mathrm{in} = h''_\mathrm{in}(p)$, $s'_\mathrm{in} = s'_\mathrm{in}(p)$, $\rho''_\mathrm{in} = \rho''_\mathrm{in}(p)$, $\rho'_\mathrm{in} = \rho'_\mathrm{in}(p)$ are functions of the cylinder pressure p.

The γ is the quality of the outgoing two-phase fluid that can be calculated by assuming that CO_2 expands from the pressure tank through the valve orifice to the cylinder isentropically. This assumption means that $s_\mathrm{z} = s_\mathrm{in}$, where s_z represents the specific enthalpy of CO_2 in front of the control valve input opening (Fig. 2).

By introducing $s_\mathrm{z} = s_\mathrm{in}$ into Eq. (16) and by rearranging yields

$$\gamma = \frac{s_\mathrm{z} - s'_\mathrm{in}}{s''_\mathrm{in} - s'_\mathrm{in}}, \tag{18}$$

The mass flow rate $\dot{m}_\mathrm{in}$ from the pressure tank through the control valve of the cross-sectional area A_o into the cylinder is given by

$$\dot{m}_\mathrm{in} = \mu A_\mathrm{o}\frac{w_\mathrm{in}}{v_\mathrm{in}}, \tag{19}$$

where μ is the discharge coefficient, it is usually assumed that the discharge coefficient is between 0.45-0.61. w_in (m/s) is the discharge flow velocity that can be defined as

$$w_\mathrm{in} = \min\left\{\sqrt{2\left(h_\mathrm{z} - h_\mathrm{in}\right)},\, a_\mathrm{sonic}\right\}, \tag{20}$$

where h_z is the specific enthalpy of CO2 in front of the control valve input opening (see Fig. 2) and a_{sonic} (m/s) is the speed of sound in the CO_2 two-phase fluid that is defined by the fluid $p-v-T$ behavior using the following formula [4]

$$a_{sonic} = \left[\rho_{12} \left(\frac{\Theta_{12}}{\rho''_{12} a_G^2} + \frac{1-\Theta_{12}}{\rho'_{12} a_{SL}^2} \right) \right]^{\left(-\frac{1}{2}\right)} \tag{21}$$

Here: $\Theta_{in} = \gamma v''_{in} / v_{in}$ accounts for the void fraction, a_G is the speed of sound in the gaseous phase and a_{SL} denotes the speed of sound in solid or liquid phase. Values of speeds of sound a_G and a_{SL} for CO_2 are also functions of pressure p.

We apply the Newton's Second Law of motion for the piston backward and forward. There are three forces acting on the piston during its moving backward: the return spring force F_{sp}, pressure force F_p and the friction force F_f.

The return spring force is generally given by the return spring constant c and the working stroke that is equal to the piston displacement x_p. It is necessary to take also the return spring preload into consideration. In this case, the return spring is initially compressed. Hence, we can express the spring force in the form

$$F_{sp} = F_{ini} + c x_p, \tag{22}$$

where the force F_{ini} is the initial compression spring force corresponding the initial spring compression, which is determined by the summation of the working and spring preload.

The pressure force F_p is given by the difference of the compression cylinder pressure and the atmospheric pressure behind the piston:

$$F_p = A_p \left(p - p_a \right). \tag{23}$$

The friction force F_f between the compression chamber internal surfaces and the contact surface of the piston seal A_f can be determined as

$$F_f = A_f f \left(p - p_a \right), \tag{24}$$

where f is the piston friction coefficient. Its value depends upon both the material, the thickness of the piston sealing and quality of the compression chamber internal surface.

The equation of motion of the piston while moving backward is

$$\left(m_p + \frac{1}{3} m_{sp} \right) \frac{\mathrm{d} v_p}{\mathrm{d}t} = F_p - F_{sp} - F_f, \tag{25}$$

where m_p and m_{sp} represent the piston and return spring mass, respectively.

There only two forces acting on the piston while moving forward: the return spring force and the friction force.

$$\left(m_p + \frac{1}{3} m_{sp} \right) \frac{\mathrm{d} v_p}{\mathrm{d}t} = -F_{sp} - F_f, \tag{26}$$

4. RESULTS OF SOLUTION

The mathematical model describing the piston dynamics includes five variables, i.e. the cylinder temperature T given by Eq.(13) , the pressure p given by Eq. (6), the total CO_2 mass m given by Eq.(4), then specific volume v given by Eq.(8) and the piston velocity is given by Eq. (25) and Eq. (26).The above-described problem was solved by numerical integration with MATLAB using the explicit fourth-order Runge-Kutta method.

Mathematical model considers a range of input data parameters and boundary conditions (i.e. the initial cylinder volume, the initial temperature, the pressure tank's specific enthalpy and entropy of CO_2, the mass of the piston, the return spring constant, the discharge coefficient, etc.). In order to present results of the solution, we selected the input data parameters and boundary conditions shown in Tab. 1 that correspond to the design parameters of the breechblock of AKM gas-operated automatic rifle [9]. Results of the solution of the developed mathematical model for this given example are presented in Fig. 3 and Fig. 4.

Table 1. Initial data parameters and boundary conditions

Quantity	**Value**	**Quantity**	**Value**
Initial cylinder volume (ml)	1.5	Return spring constant (N/m)	290
Cylinder diameter (mm)	30	Return spring mass (kg)	0.026
Piston mass (kg)	0.55	Control valve diameter (mm)	4
Piston working stroke (mm)	118	Initial cylinder temperature (K)	288
Discharge coefficient	0.7		

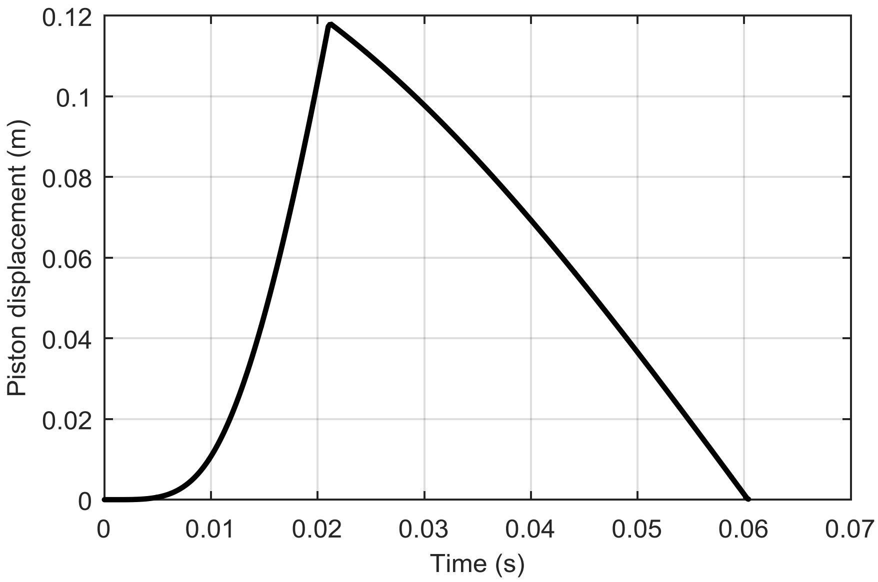

FIG. 3. Piston displacement vs. time

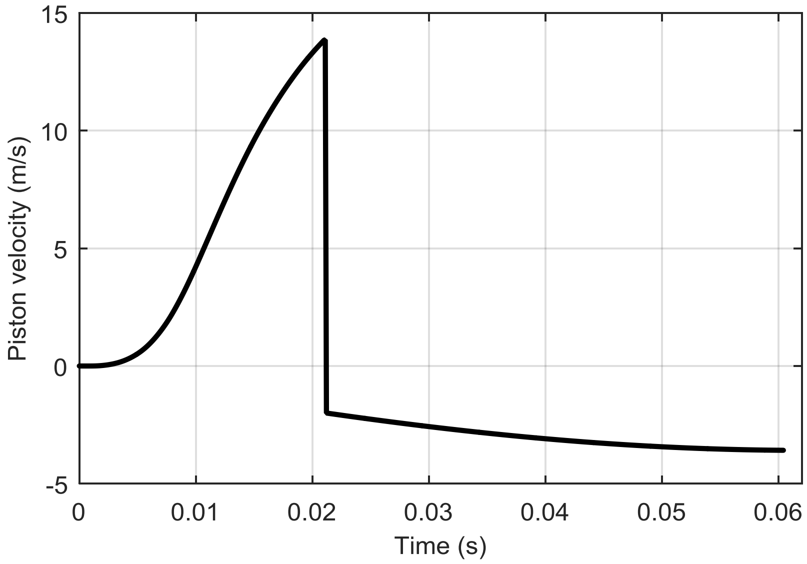

FIG. 4. Piston velocity vs. time

The dynamics of the recoil simulation device is clearly shown in Fig. 3. In the first phase of operation, the piston is accelerated backward by the pressure force. When the piston reaches its working stroke, it reaches the impact velocity $v_{\mathrm{imp/b}} = 13.85$ m/s. After the collision with the device holster, it starts moving forward by the return spring force. The impact velocity on the front position is $v_{\mathrm{imp/f}} = 3.58$ m/s (Fig. 4). The time course of the cylinder pressure in the first phase of operation is shown in Fig. 5. It can be seen that the maximum value of the pressure is 1.3 MPa.

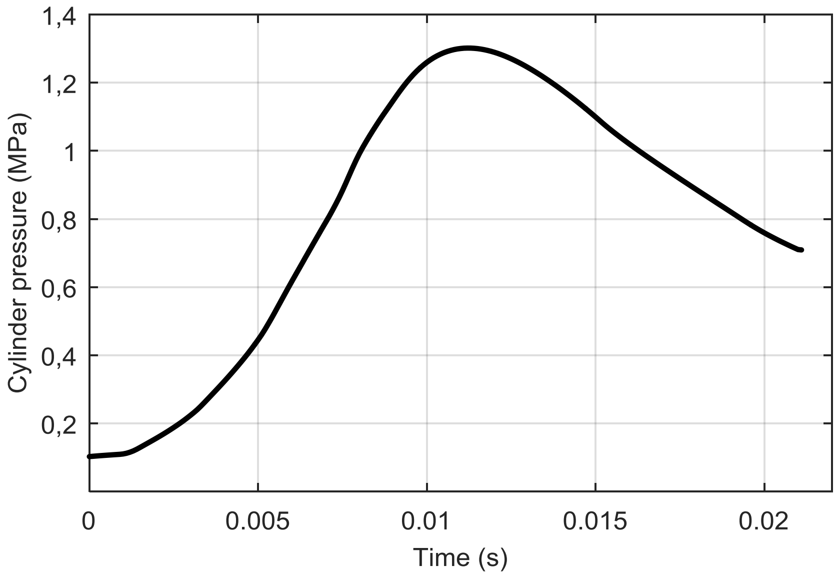

FIG. 5. Time change in cylinder pressure

5. VERIFICATION OF THE MATHEMATICAL MODEL

Results of solution of the piston impact velocity in the back position are compared with the measured values. The view of the experimental device is shown in Fig. 5.

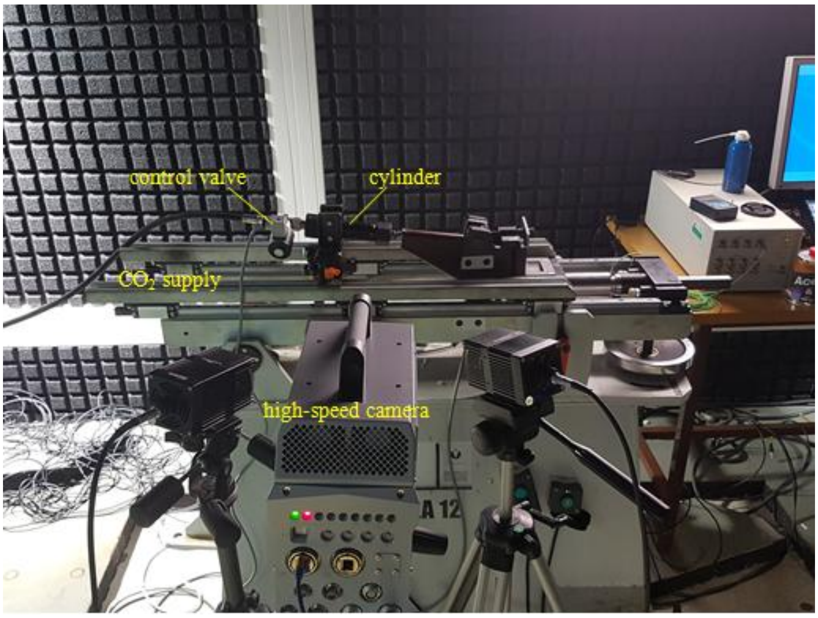

FIG. 6. View of experimental device

Here we use two types of the piston of different masses (0.309 kg and 0.240 kg). The constant of return spring is 550 N/m, the cylinder diameter is 30 mm, and the piston working stroke is 67.5 mm. A high-speed camera is used in order to measure the piston displacement. Taking differential of the measured piston displacement with respect to time yields the piston velocity.The compassion of calculated and experimentally obtained values of the piston impact velocity in the back position is shown in Tab. 2, from that we can conclude a quite good agreement between the results of the mathematical model and experimentally obtained values.

Table 2. Piston impact velocity in the back position comparison

Piston mass (kg)	Piston impact velocity in the back position (m/s)		Difference (%)
	Experiment	Model	
0.309	8.10	8.56	5.67
0.240	9.16	9.64	5.24

CONCLUSIONS

In this paper, the mathematical model for the study of the dynamics of the recoil simulation device has been formulated. The problem has been solved numerically using the Runge-Kutta method in MATLAB environment. The mathematical model is verified by the measurement of the piston impact velocity using the high-speed camera. The model provides good agreement with measured data. Obtained results enable us to analyze various influences of changes in several design parameters.

As the future improvement of the experimental device, the range of piston mass will be wider and the flow area size of the control valve can be changed.

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