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HIGH-FREQUENCY WAVES. THE EQUATION OF THE DISPLACEMENT VECTOR.

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Abstract: *In this paper we study the high frequency wave propagation and we find the displacement vector for every material point* (x_1, x_2) *of a thin plate, at time t, except for a constant factor A.*

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1. INTRODUCTION

In order to verify the accuracy of material design is usually carried out non-destructive test. This is achieved by applying a high-frequency wave propagation theory. In this paper we present the theory of high frequency wave propagation in elastic, isotropic and homogeneous, and we obtain the dispersion equation. Dispersion equation is determined from the solution of nonlinear generalized eigenvalue problem and eigenvectors.

The Lamb waves correspond to a particular case of propagation of elastic waves by an infinite solid that occur when a plate is infinite solid bounded by two parallel faces off. In this case, very often reflections of waves occur along plate faces and therefore change propagation direction. Wave propagation in this case is known where guided.

2. PROBLEM FORMULATION

We assume homogeneous and isotropic elastic plate bounded by two parallel planes located at a short distance *2h*, and want to find the vector displacement. In Figure 1 are shown the plate and axes considered. The $x_2 = \pm h$ is the free faces of the plate equations and the plane (x_1, x_2) called the sagittal plane containing the normal x_3 and the direction of wave propagation.

In the case of thin plates in which longitudinal waves *L* and vertical transverse waves *TV* are propagated in its vertical plane (x_1, x_2), with successive reflections on its free surface $\pm h$, it appears that this propagation is a coupling between the material displacements and these plane.

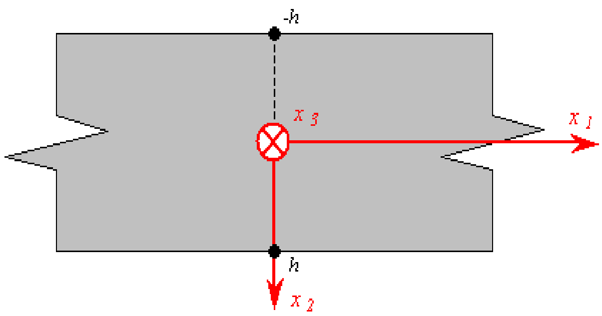

Figura 1: The coordinate axes in an isotropic and homogeneous medium

These waves are called guided waves. However, the transverse waves *TH* contained in the horizontal plane x_2 are propagated only in the horizontal plane of the plate, because their polarization is not modified by any reflections and refractions. Guided waves are also known as Lamb waves and they can be classified as follows: the first waves which are propagated in the sagittal plane (x_1, x_2) are the Lamb waves L_2 and decoupled and polarized waves are propagated in the plane (x_1, x_3) are the *TH* wave Lamb.

3. PROBLEM SOLUTION

In the following we consider the normal guided Lamb waves. These waves appear in a plate of thickness $2h$ comparable with the wavelength, due to coupling between the components longitudinal *L* and the transverse components of the wave *TV*. Thus, two types of wave Lamb can be produced. Symmetric waves depicted in Figure 2, where for each side of the middle of the plate, the longitudinal components are equal and the transverse components are opposite and antisymmetric waves from Figure 3, where for each side of the middle of the plate, the transverse components are equal and the longitudinal components are opposite.

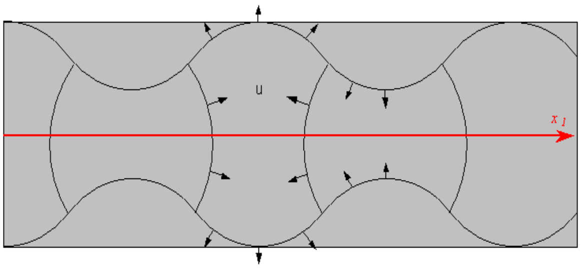

Figura 2: The symmetric Lamb waves

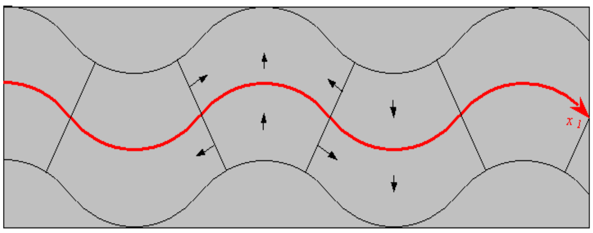

Figura 3: The antisymmetric Lamb waves

In the case of wave propagation in a continuous, isotropic and homogeneous medium, the displacement vector ***u*** of a material point can be obtained from a scalar potential φ and a vector potential ψ, so that we can write the following relation

$$\mathbf{u} = \nabla\varphi + \nabla \times \psi \tag{1}$$

In expression (1) the two potential should verify the following two equations of wave

$$\nabla^2\varphi - \frac{1}{v_L^2}\frac{\partial^2\varphi}{\partial t^2} = 0, \tag{2}$$

with

$$v_L = \left(\frac{c_{11}}{\rho}\right)^{\frac{1}{2}} = \left(\frac{\lambda + 2\mu}{\rho}\right)^{\frac{1}{2}},$$

and

$$\nabla^2\psi - \frac{1}{v_T^2}\frac{\partial^2\psi}{\partial t^2} = 0, \tag{3}$$

with

$$v_T = \left(\frac{c_{44}}{\rho}\right)^{\frac{1}{2}} = \left(\frac{\lambda}{\rho}\right)^{\frac{1}{2}},$$

where v_L and v_T are the phase velocities of the longitudinal and transversal waves, but λ and μ are the Lame's parameters. The elastic constants $c_{\alpha\beta}, \alpha, \beta = 1,2,\ldots,6$ are defined as functions of Young's modulus E and Poisson ratio ν, as follows:

$$c_{11} = c_{22} = c_{33} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}, \tag{4}$$

$$c_{12} = c_{23} = c_{13} = \frac{\nu E}{(1+\nu)(1-\nu)}, \tag{5}$$

$$c_{44} = c_{55} = c_{66} = \frac{E}{2(1+\nu)} = \frac{c_{11} - c_{12}}{2}, \tag{6}$$

in which the remaining terms $c_{\alpha\beta}$, with $\alpha \neq \beta$, are null.

Suppose the wave Lamb travels along the axis x_1 and the diffraction in x_3 is ignored. For an isotropic, elastic and homogeneous solid, the scalar and the vector potentials are trigonometric functions of time *t*, with the same frequency ω.
However, they can be expressed as follows, with the wave number *k*:

$$\begin{aligned} &\phi = \phi_0(x_2)e^{i(\omega t - kx_1)}, \\ &\psi = \psi_{0j}(x_2)e^{i(\omega t - kx_1)}, \quad j = 1,2,3. \end{aligned} \tag{7}$$

A limit wave problem can be defined of wave equations for each function of potential and boundary conditions $\sigma_{2i} = 0,\ i = 1,2,3$ on free faces $x_2 = \pm h$. For the limit this problem can have a nontrivial solution it is necessary that the frequency ω and wave number k to satisfy the following dispersion relation:

$$\frac{\tan(qh+\alpha)}{\tan(ph+\alpha)} = -\frac{4k^2pq}{\left(q^2-k^2\right)^2}, \tag{8}$$

with $\alpha = 0$ and $\alpha = \frac{\pi}{2}$, where the constants p and q are defined as follows:

$$p^2 = \frac{\omega^2}{v_L^2} - k^2, \qquad q^2 = \frac{\omega^2}{v_T^2} - k^2, \tag{9}$$

and the constant angle α can take the values 0 and $\frac{\pi}{2}$ depending on the type of symmetry of the wave.

If the relation (8) is satisfied, then we can find the potential functions, except for a constant factor and their expressions are:

$$\begin{aligned} &\phi = B\cos\left(px_2+\alpha\right)\exp\left[i\left(\omega t - kx_1\right)\right], \\ &\psi_1 = \psi_2 = 0, \\ &\psi_3 = A\sin\left(qx_2+\alpha\right)\exp\left[i\left(\omega t - kx_1\right)\right], \end{aligned} \tag{10}$$

in which constants A and B must satisfy the following system of homogeneous linear equations:

$$P\cdot\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{11}$$

where P is the following matrix

$$\begin{pmatrix} \left(k^2-q^2\right)\cos(ph+\alpha) & 2ikq\cos(qh+\alpha) \\ 2ikp\sin\left(ph+\alpha\right) & \left(k^2-q^2\right)\cos(qh+\alpha) \end{pmatrix}$$

Because now we know the functions $\varphi(x_1, x_2, t)$ and $\psi(x_1, x_2, t)$, we can apply the formula (11) and we find the displacements at time t, for any point material of the plate (x_1, x_2), except for a constant factor A, using the following expressions:

$$\begin{aligned} u_1 = qA\Big[&\cos\left(qx_2+\alpha\right) - \\ &-\frac{2k^2}{k^2-q^2}\frac{\cos\left(qh+\alpha\right)}{\cos\left(ph+\alpha\right)}\cos\left(px_2+\alpha\right)\Big]\cdot \\ &\cdot\exp\left[i\left(\omega t - kx_1\right)\right], \end{aligned} \tag{12}$$

$$\begin{aligned} u_2 = ikA\Big[&\sin\left(qx_2+\alpha\right) - \\ &-\frac{2pq}{k^2-q^2}\frac{\cos\left(qh+\alpha\right)}{\cos\left(ph+\alpha\right)}\sin\left(px_2+\alpha\right)\Big]\cdot \\ &\cdot\exp\left[i\left(\omega t - kx_1\right)\right]. \end{aligned} \tag{13}$$

3. CONCLUSIONS

In this paper we use high frequency Lamb waves in an elastic, isotropic and homogeneous medium and we find the displacement vector for every material point (x_1, x_2) of a thin plate, at time t, except for a constant factor A. This is useful in carrying out tests on homogeneity, the determination cracks or tears of a structure.

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