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AIRCRAFT JET ENGINE EXHAUST NOZZLE HYDROMECHANICAL AUTOMATIC CONTROL SYSTEM

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Abstract: *The paper deals with a hydro-mechanical automatic control system for an aircraft engine exhaust nozzle. The author identifies the system's operational behavior and establishes the motion equations, which determines the non-linear mathematical model. After its linearization and an appropriate system modification, as well as after Laplace transformation applying, one has obtained the linear non-dimensional equation system; based on it, system's block diagram and transfer functions were determined. Equation system's co-efficient were estimated based on some previous experimental determinations made by the author for a VK-1-type jet engine, as well as based on some literature existing data. One has also performed some studies concerning system's stability and some simulations (system's step input time response) in order to establish system's quality. The established simplified mathematical model, as well as the conclusions, may be useful for further similar studies.*

Keywords: *jet engine, exhaust, nozzle, control, hydraulic, actuator, model*

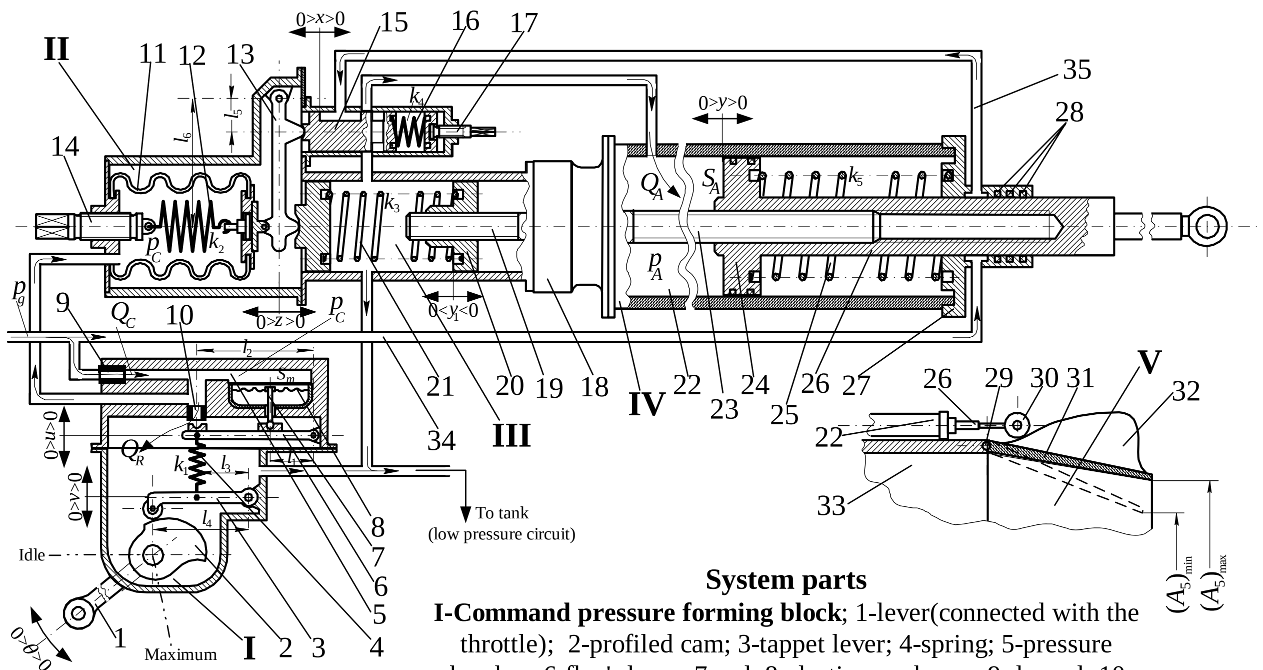
1. INTRODUCTION

Aircraft engine's exhaust nozzle is one of its most important parts, being the one responsible of the hot gases jet forming (even for some engine's thrust vectoring). Nowadays modern engines have variable-area nozzles, their area A_5 being an important control parameter during the engine operation.

Exhaust nozzle's gas-dynamic behavior and characteristics are described in [5,6,9]; meanwhile, nozzles description as controlled object is presented in [3,10]. Some possibilities for the nozzle's exhaust area automatic control (based on mechanical and/or electro-mechanical follower systems) were presented in [3,9] and, by the author, in [2,4]. Nozzle's exhaust area must be commanded with respect to some other control parameters, such as injection fuel flow rate, turbine's (or compressor's) pressure ratio, engine's speed, throttle's position etc.

The purpose of this paper is to identify the nozzle's opening system in fig. 1 as a controlled object, by describing it through its mathematical model, as well as through its transfer function and by performing some stability and quality studies, which can be further extended for a whole class of similar systems, or can be used for other specific engine control embedded systems studies.

The system in figure 1 consists of the following main parts: I-command pressure signal forming block; II-slide-valve command block; III-position transducer (feed-back); IV-hydraulic actuator; V- exhaust nozzle with mobile flaps.



System parts

I-Command pressure forming block; 1-lever(connected with the throttle); 2-profiled cam; 3-tappet lever; 4-spring; 5-pressure chamber; 6-flap's lever; 7-rod; 8-elastic membrane; 9-drossel; 10-nozzle drossel; II-Slide-valve's command block; 11-silphon; 12-silphon's spring; 13-slide-valve's lever; 14-adjusting bolt;15-slide-valve; 16-slide-valve's spring; 17-adjusting bolt; **III-Position transducer**; 18-planetary reducing gear; 19-screw with thread on the left; 20-nut; 21-transducer's spring; **IV-actuator**; 22-hydraulic cylinder; 23-screw with thread on the right; 24-piston with nut; 25-actuator's spring; 26-actuator's rod; 27-lid; 28-sealing rings; **V-exhaust nozzle**; 29-knuckle; 30-pressing roll; 31-volet (flap); 32-profiled contour; 33-exhaust tube; 34,35-supplying pipes;

Figure 1. System constructive and operating scheme

As fig. 1 shows, the system works with respect to the throttle's position (through the 1-lever rotation angle θ); nozzle's flaps outer contours (32) are profiled, so the actuator's rod (26) displacement presses the 30-roll on this contour, determining nozzle flaps' positioning. Consequently, nozzle's exhaust area A_5 is determined by the throttles position, so this system is a follower one. Nozzle's opening command law $A_5 = A_5(\theta)$ is given by the 2-cam profile, cam connected to the 1-lever. System's hydraulic actuator has one active chamber A, its rod position being established by the balance between the pressure force in chamber A and the 25-spring's elastic force. The active pressure p_A is determined by the 15-slide-valve's position *x*, which itself is the result of a force balance (the pressure force of the silphon 12, given by the command hydraulic pressure p_C and the actuator's position transducer 21-spring's elastic force).

System's supply is assured by a hydraulic pump (which belongs to the aircraft or to the engine). Consequently, hydraulic fluid's flow rate and pressure are given by the pump's speed, therefore system supplying hydraulic pressure p_g is usually assured by a constant pressure valve mounted after the pump

According to these observations, one can affirm that exhaust area A_5 depends on the actuator's rod displacement y, which itself depends on the throttle's angle θ and on the supplying pressure p_g

$$A_5 = A_5(y); y = y(\theta, p_g). \qquad (1)$$

2. SYSTEM MATHEMATICAL MODEL

The studied system's mathematical model consists of the motion equation for each of its parts. The non-linear equation will be transformed, in order to bring them to an acceptable form for further studies, as well as for simulations.

2.1 Non-linear mathematical model

The system non-linear motion equations are the following:

a) command pressure forming block

$$v = v(\theta), \tag{2}$$

$$k_1\left(u + \frac{l_3}{l_4}v\right)l_2 = p_C S_m l_{1,} \tag{3}$$

$$Q_C = \mu_g \frac{\pi d_g^2}{4}\sqrt{\frac{2}{\rho}}\sqrt{p_g - p_C}\ , \tag{4}$$

$$Q_R = \mu\ \pi d_s u\sqrt{\frac{2}{\rho}}\sqrt{p_C}\ , \tag{5}$$

$$Q_C - Q_R = \beta V_{C0}\frac{\mathrm{d}p_C}{\mathrm{d}t} + S_m\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{l_1}{l_2}u\right), \tag{6}$$

b) slide-valve command block

$$\begin{aligned}&[S_C p_C - k_2(z + z_r)]l_6 - k_2(z + y_1)l_6 = \\ &= l_5 k_4(x + x_r),\end{aligned} \tag{7}$$

$$x = \frac{l_5}{l_6}z\ , \tag{8}$$

c) actuator's rod displacement transducer

$$y_1 = i_{tr}y;\ , \tag{8}$$

d) actuator

$$Q_A = \mu_b bx\sqrt{\frac{2}{\rho}}\sqrt{p_g - p_A}\ , \tag{9}$$

$$Q_A = \beta V_{A0}\frac{\mathrm{d}p_A}{\mathrm{d}t} + S_A\frac{\mathrm{d}y}{\mathrm{d}t}, \tag{10}$$

$$\begin{aligned}S_A p_A = m_p\frac{\mathrm{d}^2(y + y_s)}{\mathrm{d}t^2} + \xi\frac{\mathrm{d}(y + y_s)}{\mathrm{d}t} + \\ + k_5(y + y_s),\end{aligned} \tag{11}$$

where v is the 3-lever displacement, depending on 2-cam's profile; $u-$nozzle-flap system's 6-lever displacement; $l_1, l_2, l_3, l_4, l_5, l_5 -$levers' arms' length; $S_m -$membrane 8 surface area; $Q_C, Q_R -$hydraulic fluid flow rates from/into the command pressure forming block; $\mu_g, \mu, \mu_b -$flow rate co-efficient; $\rho-$fluid's density; $\beta-$compressibility co-efficient; $d_g, d_s -$drossels (9 and 10) diameters; k_1, k_2, $k_3, k_4, k_5 -$ springs' elastic constants; $V_{C0} -$11-silphon's volume; $S_C -$11-silphon's lid surface area; $z-$13-lever displacement; $z_r -$ pre-tensioning displacement of the 12-spring; $x_r -$ pre-tensioning displacement of the 16-spring; $i_{tr} -$18-gear reduction ratio; $y_1 -$transducer's piston's displacement; $S_A -$actuator piston surface area; $m_p -$piston mass; $\xi-$friction co-efficient; $V_{A0} -$ actuator active chambers volume; $b-$slide-valve slot width; $y_s -$instant piston's position.

The above determined non-linear equation system (equations (2) to (11)) is difficult to be used for further studies, so it can be brought to a linear form, using the small perturbation method, considering formally any variable or parameter X as

$$X = X_0 + \Delta X, \tag{12}$$

where $X_0 -$ steady state regime's value, $\Delta X -$ parameter's deviation and $\overline{X} = \dfrac{\Delta X}{X_0}$ the non-dimensional deviation.

2.2 Non-dimensional linearised model

In order to determine a linearised form for the above equation system, one has to identify the main parameters, which are θ, v, u, z, x, y, $y_1, Q_C, Q_R, Q_A, p_C, p_A, p_g$. The displacements x_r, z_r (representing the adjustments made during the ground testing period), has no relevance for the system dynamic behavior, so they are excluded.

Expressing each one of them as in eq. (12) and introducing them into the equation system, after eliminating the terms containing the steady state regime, one obtains the system's linear form.

Using some appropriate chosen amplifying terms, the linearised mathematical model can be transformed in a non-dimensional one; after applying the Laplace transformation, one obtains the system's linear non-dimensional mathematical model, as follows

$$\overline{v} = k_\theta\overline{\theta}, \tag{13}$$

$$\overline{u} = k_{uc}\overline{p}_C - k_{uv}\overline{v}, \tag{14}$$

$$(\tau_C \mathrm{s} + 1)\overline{p}_C = k_u(\tau_u \mathrm{s} + 1) - k_{gp}\overline{p}_g, \tag{15}$$

$$\overline{x} = k_{xc}\overline{p}_C - k_{xy}\overline{y}, \tag{16}$$

$$(\tau_A \mathrm{s} + 1)\overline{p}_A = k_{gA}\overline{p}_g + k_x\overline{x} - \tau_y \mathrm{s}\overline{y}, \tag{17}$$

$$\bar{y} = \frac{k_{yA}}{T_y^2 \mathrm{s}^2 + 2\omega_0 T_y \mathrm{s} + 1}\bar{p}_A, \tag{18}$$

together with $\bar{A}_5 = k_{5y}\bar{y}$, where the involved co-efficient are

$$k_\theta = \frac{\theta_0}{v_0}\left(\frac{\partial v}{\partial \theta}\right)_0,\ k_{cg} = \frac{\mu_g \pi d_g p_{g0}}{4Q_{C0}\sqrt{2\rho\left(p_{g0} - p_{C0}\right)}},$$

$$k_{uc} = \frac{S_m l_1 p_{C0}}{k_1 l_2 u_0},\ k_{CC} = \frac{\mu_g \pi d_g^2 p_{g0}}{4Q_{C0}\sqrt{2\rho\left(p_{g0} - p_{C0}\right)}},$$

$$k_{uv} = \frac{l_3 v_0}{l_4 u_0},\ k_{Rc} = \frac{\mu \pi d_s \sqrt{p_{C0}}}{Q_{C0}\sqrt{2\rho}},\ \tau_A = \frac{\beta V_{A0} p_{A0}}{Q_{A0} k_{AA}}$$

$$k_{Ru} = \frac{\mu \pi d_s u_0 \sqrt{2 p_{C0}}}{Q_{C0}\sqrt{\rho}},\ \tau_{cc} = \frac{\beta V_{C0} p_{C0}}{Q_{C0}},$$

$$k_{CRC} = k_{CC} + k_{Rc},\ \tau_{uc} = \frac{S_m u_0 l_1}{Q_{C0} l_2}, \tau_y = \frac{S_A y_0}{k_{AA} Q_{A0}},$$

$$k_{xz} = \frac{z_0 l_6 \left(k_2 + k_3\right)}{k_4 x_0 l_5}, k_{xy} = \frac{y_0 l_6 k_3 i_{tr}}{k_4 x_0 l_5}, \tag{19}$$

$$k_{Ag} = \frac{\mu_b b x_0 p_{g0}}{Q_{A0}\sqrt{2\rho\left(p_{g0} - p_{A0}\right)}},\ k_{xC} = \frac{S_C p_{C0} l_6}{k_4 x_0 l_5},$$

$$k_{AA} = \frac{\mu_b b x_0 p_{A0}}{Q_{A0}\sqrt{2\rho\left(p_{g0} - p_{A0}\right)}},\ k_{Ay} = \frac{S_A p_{A0}}{k_5 y_0},$$

$$k_{gA} = \frac{k_{Ag}}{k_{AA}}, k_{xA} = \frac{k_{Ax}}{k_{AA}}, k_x = \frac{1}{1 + k_{xz}}.$$

Based on some practical observation, one can make some supplementary hypothesis, as follows: a) the fuel is a non-compressible fluid ($\beta = 0$), so the terms containing it become null ($\tau_A = \ = \tau_C = 0$); b) the inertial effects are very small, so the term containing m_p could be considered also null. Consequently, eqs. (15) and (17) become

$$\bar{p}_C = k_u\left(\tau_u \mathrm{s} + 1\right) - k_{gp}\bar{p}_g, \tag{20}$$

$$\bar{p}_A = k_{gA}\bar{p}_g + k_x \bar{x} - \tau_y \mathrm{s}\bar{y}. \tag{21}$$

Based on the simplified model, one can build the system's block diagram with transfer functions, which is depicted in figure 2.

2.3 System transfer function

As fig. 2 shows, the system has two inputs ($\bar{\theta}$ and $\bar{p}_g$) and a single output ($\bar{y}$). Using the above-determined mathematical model's equations, one can obtain an equivalent form:

$$\bar{y} = \frac{\left(\tau_{u\theta}\mathrm{s} + \rho_\theta\right)\bar{\theta} + \left(\tau_{ug}\mathrm{s} + \rho_g\right)\bar{p}_g}{\left(\tau_{yA}\mathrm{s} + 1\right)\left(\tau_{uA}\mathrm{s} + 1\right)}, \tag{22}$$

where

$$\tau_{yA} = \frac{k_{yA}\tau_y + \dfrac{\xi}{k_5}}{1 + k_{yA}k_x k_{xy}},\ \tau_{uA} = \frac{k_u k_{uc}\tau_u}{k_u k_{uc} - 1},$$

$$\tau_{u\theta} = \frac{k_{yA}k_x k_{xc} k_u k_{uv} k_\theta \tau_u}{\left(k_u k_{uc} - 1\right)\left(1 + k_{yA}k_x k_{xy}\right)},$$

$$\rho_\theta = \frac{k_\theta - k_{yA}k_{gA}k_x k_{xc} k_{uc} k_{uv}}{\left(k_u k_{uc} - 1\right)\left(1 + k_{yA}k_x k_{xy}\right)}, \tag{23}$$

$$\tau_{ug} = \frac{k_{yA}k_{gA}k_u k_{uc}\tau_u}{\left(k_u k_{uc} - 1\right)\left(1 + k_{yA}k_x k_{xy}\right)},$$

$$\rho_g = \frac{k_{yA}k_{gA}\left(k_u k_{uc} - 1\right) + k_{yA}k_x k_{xc} k_{gp}}{\left(k_u k_{uc} - 1\right)\left(1 + k_{yA}k_x k_{xy}\right)}.$$

It results two transfer functions:

a) with respect to the throttle's position

$$H_\theta(\mathrm{s}) = \frac{\tau_{u\theta}\mathrm{s} + \rho_\theta}{\left(\tau_{yA}\mathrm{s} + 1\right)\left(\tau_{uA}\mathrm{s} + 1\right)}, \tag{24}$$

b) with respect to the supplying pressure

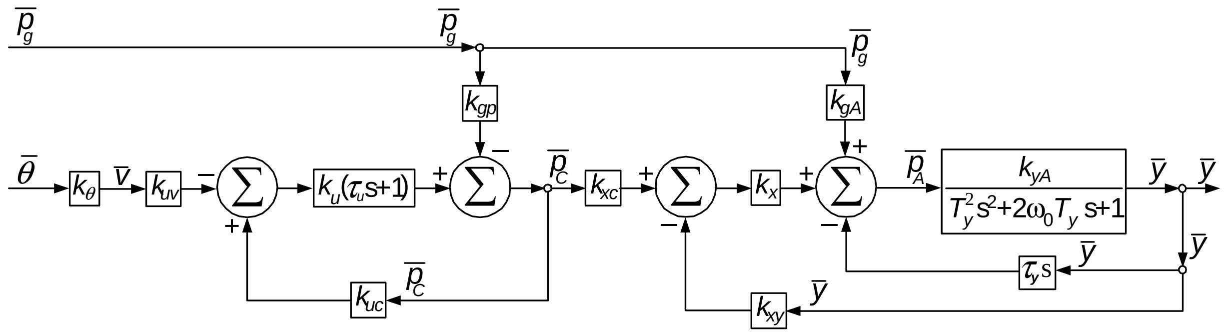


Figure 2. System block diagram with transfer functions

$$H_g(s) = \frac{\tau_{ug}s + \rho_g}{(\tau_{yA}s + 1)(\tau_{uA}s + 1)}. \qquad (25)$$

Usually, the supply pressure p_g can be assumed as constant, because it is assured by a hydraulic pump (spinned up by the engine's rotor) and a constant pressure valve. Consequently, one can assume that $\overline{p}_g = 0$, which means that only the first transfer function, $H_\theta(s)$, becomes relevant.

3. SYSTEM STABILITY AND QUALITY

Transfer functions (24) and (25) have their characteristic polynomial $(\tau_{yA}s + 1)(\tau_{uA}s + 1)$ of second order; furthermore, one can observe that, as long as τ_{yA} and τ_{uA} are strictly positives, the characteristic polynomial has real negative roots, that means that the system is a stabile one.

While τ_{yA} is always positive, because all the factors involved in its formula (see eq.(23)) are strictly positives, the other time constant τ_{uA} should be studied, because of its denominator. Consequently, the condition that τ_{uA} were positive is

$$k_u k_{uc} - 1 > 0. \qquad (26)$$

Based on the expressions (19) for k_u and k_{uc}, one obtains (27)

$$\frac{8\mu d_s p_{C0}\sqrt{p_{g0} - p_{C0}}}{\mu d_g^2\sqrt{p_{C0}} + 4\mu_s d_s u_0 \sqrt{p_{g0} - p_{C0}}} \cdot \left(\frac{l_1}{l_2}\right)\frac{1}{k_1} > 1,$$

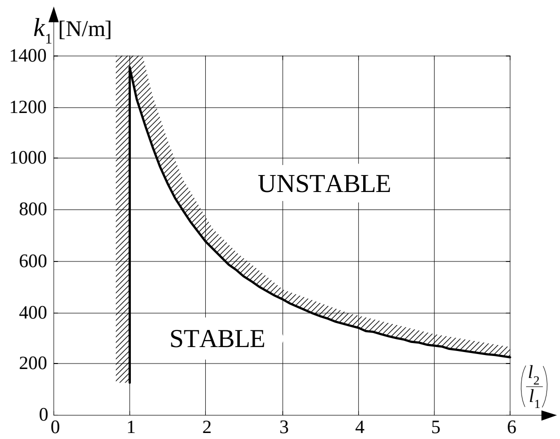


Figure 3. System's stability domain

which leads to a relation between the command pressure forming block's 4-spring elastic constant k_1 and the 6-lever arms' ratio. So, one can choose the elastic constant with respect to the lever arms ratio in order to obtain system's stability. Figure 3 shows the co-relation between those two parameters, as well as the stability/instability domains limits.

Regarding system's quality, one has performed some simulations (based on the block diagram in fig. 2), studying system's step response (more precisely: system response for step input of a single input parameter, $\overline{\theta}$, respectively for both input parameters $\overline{\theta}$ and $\overline{p}_g$). For the co-efficient presented in (19)-formulas calculation one has considered data from a VK-1F jet engine. One has obtained a particular form for eq. (22) and, considering also the co-relation to $\overline{A}_5$ [2], it has resulted as follows

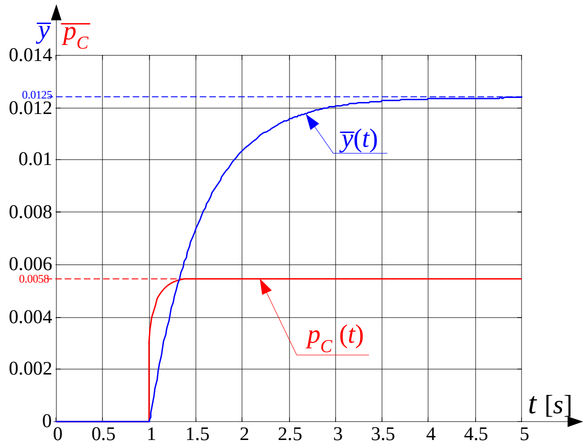


Figure 4. System's step response for $\overline{\theta}$ input

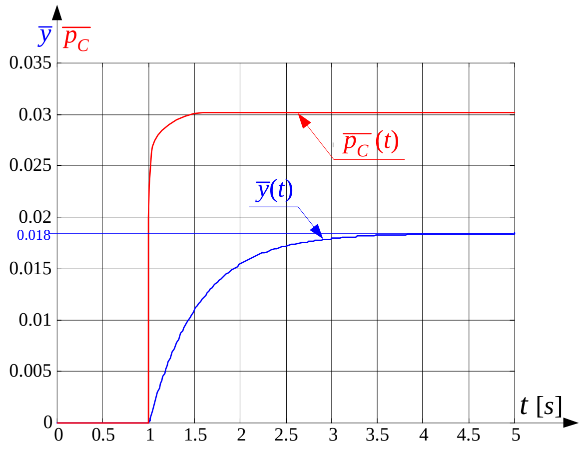


Figure 5. Step response for $\overline{\theta}$ and $\overline{p}_g$ inputs

$$\bar{A}_5(t) = \frac{0{,}0712\times(0{,}0422\mathrm{s}+0{,}1025)}{(0{,}1474\mathrm{s}+1)(0{,}6376\mathrm{s}+1)}\bar{\theta}(t) + \\ + \frac{0{,}0453\times(0{,}0198\mathrm{s}+0{,}2556)}{(0{,}1474\mathrm{s}+1)(0{,}6376\mathrm{s}+1)}\bar{p}_g(t). \qquad (28)$$

System step response for a single input parameter is depicted in figure 4 and for two input parameters – in figure 5. As output parameters one has considered, obviously, the actuator's rod displacement $\bar{y}$, but also the command pressure $\bar{p}_C$.

4. CONCLUSIONS

The studied hydro-mechanical exhaust nozzle's opening system is a second order control system, as its transfer function shows.

Transfer functions characteristic polynomial has two real roots, one strictly negative, the other one negative under an explicit condition (see eq. (26)), which determines both a relation between the k_1 spring elastic constant and the lever arms ratio (27), and the stability domains too.

System's quality studies reveal an aperiodic stability for both analyzed situation and for both output parameters. The system is a static one, its stabilization being fulfilled with static error.

One can observe that rod's displacement parameter $\bar{y}$ stabilizes itself in 2.5…3 seconds, with a static error of 1.25% (for a single input) until 1.8% (for two inputs). Command pressure's parameter $\bar{p}_C$ is more rapid, stabilizing in 0.5 seconds, with a static error between 0.58% (for a single input) and 3.0% (for two inputs).

The influence of the throttle displacement parameter $\bar{\theta}$ above the rod's displacement parameter is bigger than supply pressure's parameter $\bar{p}_g$ influence. Obviously, pressure's parameter $\bar{p}_g$ influence above the command pressure $\bar{p}_C$ parameter is bigger, as the step responses diagrams reveals.

Formulas (22) for $\bar{y}$ and (28) for $\bar{A}_5$ may be considered the simplified mathematical model for the above described system and can be used for further studies involving embedded engine control systems where such an exhaust nozzle control system (or similar) is used.

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