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MATHEMATICAL ESTIMATION IN FINANCIAL ECONOMICS

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Abstract: *This paper is an application to the euro-leu evolution and a prediction for this evolution. We are testing if that euro-leu exchange rate fluctuations can be approximated by a normal distribution.*

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1. Introduction

In this lecture we will derive the formulas for the symmetric two-sided prediction interval for the n + 1-st observation and the upper-tailed prediction interval for the n+1-st observation from a normal distribution when the variance S^2 is unknown. We will need the following theorem from probability theory that gives the distribution of the statistic $X - X_n + 1$.

Suppose that $X_1, X_2, \ldots, X_n, X_{n+1}$ is a random sample from a normal distribution with mean μ and variance S^2.

Theorem 1. The random variable $T = (X - X_{n+1})/(q_n + \ln S)$ has a distribution with n − 1 degrees of freedom.

2. The two-sided prediction interval formula

Now we can prove the theorem from statistics giving the required prediction interval for the next observation x_n+1 in terms of n observations $x_1, x_2, \cdots, x_n$. Note that it is symmetric around X. This is one of the basic theorems that you have to learn how to prove. There are also asymmetric two-sided prediction intervals.

Theorem 2. The random interval

$$\overline{X} - \chi_{\frac{\alpha}{2},n-1} S\sqrt{1+\frac{1}{n}}, \overline{X} + \chi_{1-\frac{\alpha}{2},n-1} S\sqrt{1+\frac{1}{n}}$$

is a 100(1-α)% prediction interval for X_{n+1}.

In the next theorem we will give the formula for the upper-tailed prediction interval for the next observation X_{n+1}.

Theorem 3. The random interval

$$(\overline{X} - \chi_{\frac{\alpha}{2},n-1} S\sqrt{1+\frac{1}{n}}, \infty)$$

is a 100(1-α)% prediction interval for the next observation X_{n+1}.

Once we have an actual sample $x_1, x_2, \ldots, x_n$, we obtain the observed value

$(\overline{x}-\chi_{\frac{\alpha}{2},n-1}S\sqrt{1+\frac{1}{n}},\infty)$ of the upper-tailed prediction interval $(\overline{X}-\chi_{\frac{\alpha}{2},n-1}S\sqrt{1+\frac{1}{n}},\infty)$.

The observed value of the upper-tailed prediction interval is also called the upper-tailed 100(1-α)% prediction interval for x_{n+1}.

The number random variable $\overline{X}-\chi_{\frac{\alpha}{2},n-1}S\sqrt{1+\frac{1}{n}}$

or its observed value $\overline{x}-\chi_{\frac{\alpha}{2},n-1}S\sqrt{1+\frac{1}{n}}$ is often called a prediction lower bound for x_{n+1} because

$$P(\overline{X}-\chi_{\frac{\alpha}{2},n-1}S\sqrt{1+\frac{1}{n}}<X_{n+1})=1-\alpha.$$

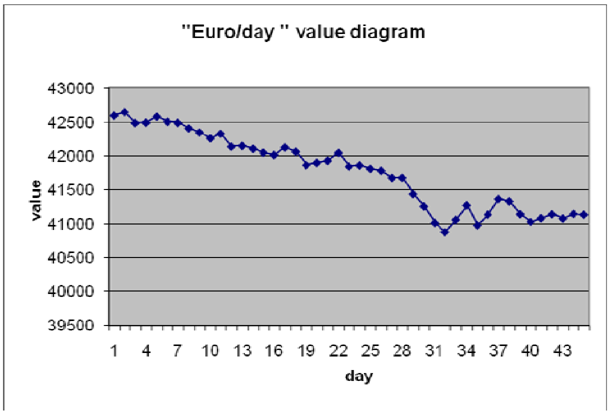
3. Application

Choose the euro-dollar of the last 45 days: 42600, 42649, 42490, 42496, 42584, 42509, 42493, 42412, 42350, 42261, 42331, 42139, 42150, 42108, 42051, 42016, 42127, 42065, 41869, 41902, 41932, 42048, 41848, 41865, 41816, 41788, 41683, 41685, 41439, 41260, 41020, 40881, 41065, 41276, 40984, 41141, 41367, 41333, 41147, 41035, 41089, 41146, 41084, 41152, 41140.

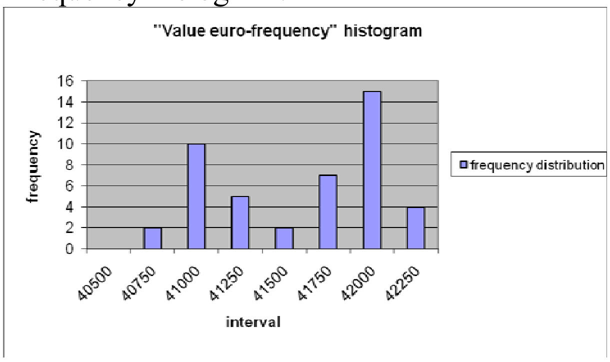
Using these data we treat the following problems:

1. Graphical Representation

a) determining the change depending euro-leu on the day:



b) dividing the interval [40881;42649] in subintervals and calculating the frequencies of each subinterval as we determine the frequency histogram:

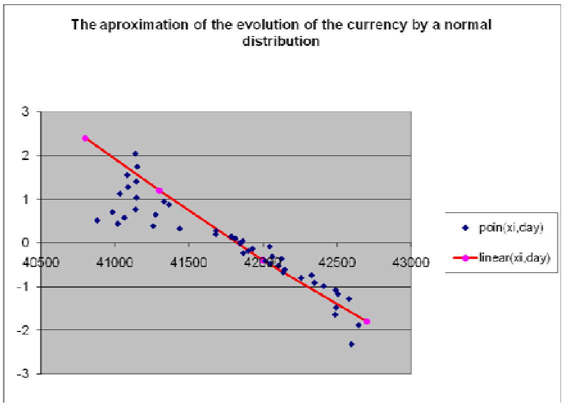


2. Testing that euro-leu exchange rate fluctuations can be approximated by a normal distribution.

To solve the problem we will sort the data ascending xi, we will plot coordinate points (x_i, z_i) ,1,..,45 , where z_i are standardized normal scores given by:

$$\frac{i-0.5}{n}=P(X<x_i)=\Phi(x_i),$$

and $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}e^{-\frac{t^2}{2}}$ is the normal distribution $N(0,1)$. So we obtain:



It is noted that the points are located approximately on a straight line and therefore the data hold normal distribution.

3. Numerical characteristics determination:

The selection mean: $\overline{X} = \dfrac{x_1 + \ldots + x_n}{n}$

$$\overline{X} = 41773.91$$

The standard deviation:

$$S^2 = \frac{\left(x_1 - \overline{X}\right)^2 + \ldots + \left(x_n - \overline{X}\right)^2}{n-1}$$

$$S^2 = 302619.6$$

so , $S = \sqrt{\dfrac{\left(x_1 - \overline{X}\right)^2 + \ldots + \left(x_n - \overline{X}\right)^2}{n-1}}$

$$S = 550.1087$$

4. We determine an interval of:

i) 100(1-α)% confidence for the mean μ, in the cases α=0.50, 0.75, 0.95, and 0.98.

ii) 100(1-α)% confidence for the deviation σ^2, in the cases α=0.50, 0.75, 0.95, and 0.98.

i)

Interval of 50% confidence for mean estimation	Interval of 75% confidence for mean estimation	Interval of 95% confidence for mean estimation	Interval of 98% confidence for mean estimation
[41678; 41869]	[41845; 41902]	[41583; 41964]	[41553; 41994]

ii)

Interval of 50% confidence for standard deviation estimation	Interval of 75% confidence for standard deviation estimation	Interval of 95% confidence for standard deviation estimation	Interval of 98% confidence for standard deviation estimation
[484;647]	[440;722]	[377;877]	[352;962]

Note that with increasing confidence we observe the increase of the length of the interval.

5. We determine a prediction interval for the random variable X_{n+1} of the next day currency

We have $Z = \dfrac{X_{n+1} - \overline{X}}{S\sqrt{1 + \frac{1}{n}}}$, a random variable with n-1 freedom degree,

$$P\left(-\chi_{\frac{\alpha}{2},n-1} \le Z \le \chi_{1-\frac{\alpha}{2},n-1}\right) = 1 - \alpha$$

so $-\chi_{\frac{\alpha}{2},n-1} \le \dfrac{X_{n+1} - \overline{X}}{S\sqrt{1 + \frac{1}{n}}} \le \chi_{1-\frac{\alpha}{2},n-1}$.

We have

$$\overline{X} - \chi_{\frac{\alpha}{2},n-1} S\sqrt{1 + \frac{1}{n}} \le X_{n+1} \le \overline{X} + \chi_{1-\frac{\alpha}{2},n-1} S\sqrt{1 + \frac{1}{n}}$$

Prediction interval for α=0,50	Prediction interval for α=0,50	Prediction interval for α=0,50	Prediction interval for α=0,50

[41125; 42422]	[40904; 42643]	[40483; 43064]	[40276; 43271]

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